Irrigation Canal: Optimization

Calculus I Project

The purpose of this project is to use the calculus concepts to design an irrigation canal that will minimize cost given various constraining conditions. The figure shows the cross section of an irrigation canal that is to be constructed in the shape of a trapezoid.

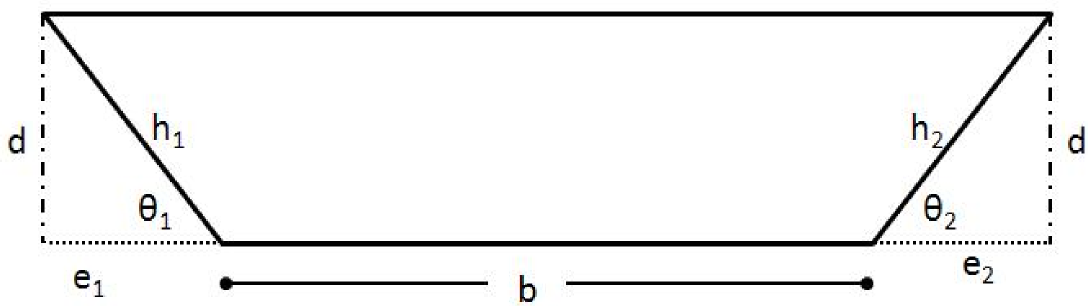

Figure 1: Cross section of canal in the shape of a trapezoid. The cross sectional area A is to be fixed to maintain fixed fluid flow.

To maintain a fixed flow rate, the cross sectional area A is to be fixed. The three sides of the trapezoid (all sides except for the top) are to be lined with concrete with a specified thickness so that the cost of construction is determined by the total length L of the concrete to be used. The value of L depends on the cross sectional area A as well as the choice of the depth d and the angles θ_1 and θ_2 as shown in the diagram. **Regardless of the appearance of the figure, we will not assume in general that the angles θ_1 and θ_2 are equal—hence we will not assume in general that $h_1 = h_2$ or that $e_1 = e_2$.**

Consider Various Optimization Scenarios

A. Consider the simple case that the trapezoid is isosceles so that $\theta_1 = \theta_2 = \theta$. Suppose that the cross sectional area $A = 80$ sq. ft., and obtain a formula for the length L of concrete in terms of the two parameters d and θ. (Note that none of b, h_1, h_2, e_1 or e_2 should appear in your final formula for L. They may appear in the formula as you are constructing it, but should be replaced with expressions in d and the two angles.)

B. Still under the conditions in part A., show that if d is considered constant, that the length L as a function of θ alone takes its absolute minimum value when $\theta = \frac{\pi}{3}$ (remember that θ must be acute here). Be certain to justify the claim that the minimum is absolute.

C. Continue to assume that $A = 80$ sq. ft. If θ is fixed, and d is allowed to be a variable, show that the absolute minimum of L as a function of d alone occurs when

$$d = \sqrt{\frac{80 \sin\theta}{2 - \cos\theta}}.$$

Again be certain to justify the claim that the minimum is absolute. (Remember that θ is considered constant here, we just leave it as θ because we don't know what its constant value is.)

D. Assume that the cross sectional area is still $A = 80$ sq. ft. Now drop the assumption that $\theta_1 = \theta_2$. Find a formula for the length L of the concrete in terms of the three parameters d, θ_1, and θ_2. (Again, none of b, h_1, h_2, e_1 or e_2 should appear in your final formula for L.)

Find the formula if a number for A is not specified—i.e. just letting A appear in the formula.

E. Suppose that due to the bedrock, the angle $\theta_1 = \frac{\pi}{6}$ must be used. If d is fixed, find the value of θ_2 that produces the absolute minimum value of L as a function of θ_2 alone. How does this compare to what you found in part B? Can you make and justify a conjecture about the optimum value for θ_2 for any fixed value of θ_1?

F. Now assume that both θ_1 and θ_2 are fixed. They aren't necessarily equal, but they are unknown constants between 0 and $\frac{\pi}{2}$. Determine the value of d that gives the absolute minimum value of L as a function of d alone. (As is part C. it is expected that d will have a formula that depends on both θ_1 and θ_2. So don't be worried about that. Just express d in terms of these other values.)