Solutions to Review for Exam I

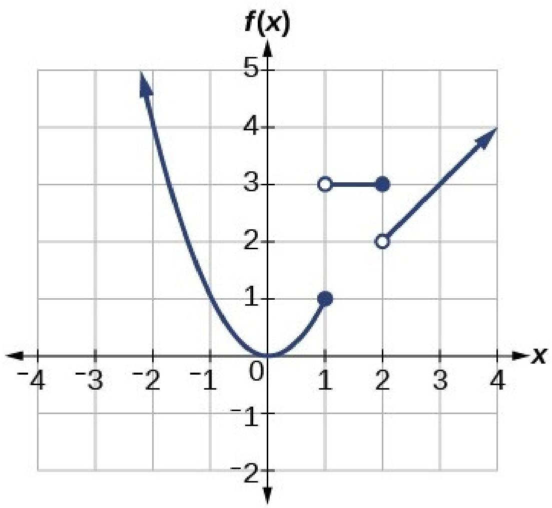
MATH 1190

Sections Covered: 1.1, 1.2, 1.3, 1.4, 1.5, 2.1

These solutions are NOT detailed.

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Use the graph of $y = f(x)$ shown to answer the following questions.



1. Evaluate if possible $\lim_{x\to 1^-} f(x) = 1$
2. Evaluate if possible $\lim_{x\to 1^+} f(x) = 3$
3. Evaluate if possible $\lim_{x\to 1} f(x)$ DNE (one sided limits disagree)
4. Evaluate if possible $f(1) = 1$
5. Evaluate if possible $\lim_{x\to 3} f(x) = 3$
6. Evaluate if possible $\lim_{x\to 2^+} f(x) = 2$
7. Is f continuous from the left at 1? Yes since $\lim_{x\to 1^-} f(x) = f(1)$.
8. Is f continuous from the right at 1? No since $\lim_{x\to 1^+} f(x) \neq f(1)$.
9. Does f have a removable discontinuity at 2? No, it is a jump as opposed to a hole.

(2) Evaluate each limit if possible using limit laws.

(a) $\lim_{x \to 4} \frac{x^2 - 16}{x^2 - 2x - 8} = \frac{4}{3}$

(b) $\lim_{t \to 0} \frac{e^{3t}}{t + 1} = 1$

(c) $\lim_{\theta \to \frac{\pi}{2}} (\cos\theta - \sin\theta) = -1$

(d) $\lim_{x \to 3} \frac{\sqrt{4 - x} - 1}{x - 3} = -\frac{1}{2}$

(e) $\lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = -\frac{1}{4}$

(3) Let $f(x) = \sqrt{x}$. (a) Set up the ratio $\frac{f(x)-f(1)}{x-1}$. Then use limit laws and any necessary algebra to evaluate the limit

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} = \lim_{x \to 1} \left(\frac{\sqrt{x} - 1}{x - 1} \right) \left(\frac{\sqrt{x} + 1}{\sqrt{x} + 1} \right) = \cdots = \frac{1}{2}$$

(4) Determine whether the given function is continuous at the indicated point c . Justify your claims.

(a) $f(x) = \begin{cases} \frac{\sin x}{2x}, & x \neq 0 \\ \frac{1}{2}, & x = 0 \end{cases}$ $\quad c = 0$ Yes since $\lim_{x \to 0} f(x) = \frac{1}{2} = f(0)$.

(b) $f(x) = \begin{cases} (x - 1)^2, & x \leq 1 \\ \tan\left(\frac{\pi x}{4}\right), & x > 1 \end{cases}$ $\quad c = 1$ No since $\lim_{x \to 1^-} f(x) = 0,\ \lim_{x \to 1^+} f(x) = 1,\ \lim_{x \to 1} f(x)$ DNE

(5) Evaluate each limit using appropriate limit statements.

(a) $\lim_{x \to 0} \frac{\sin(2x)}{\sin(3x)} = \frac{2}{3}$

(b) $\lim_{t \to 0} 2t \csc(4t) = \frac{1}{2}$

(c) $\lim_{\theta \to 0} \frac{\cos(2\theta)}{\cos(7\theta)} = 1$

(6) Evaluate each limit if possible. If a limit is ∞ or $-\infty$, give the appropriate infinity as the answer. If the limit doesn't exist, just state that it DNE with some justification.

(a) $\displaystyle \lim_{x\to 3^-} \frac{1-x}{x-3} = \infty$

(b) $\displaystyle \lim_{t\to 0} \frac{1}{|t|} = \infty$

(c) $\displaystyle \lim_{\theta\to \pi^+} \tan\left(\frac{\theta}{2}\right) = -\infty$

(d) $\displaystyle \lim_{x\to 0} \csc x$ DNE (goes to $+\infty$ on the right and $-\infty$ on the left)

(7) Evaluate each limit at infinity. If it doesn't exist, justify this claim.

(a) $\displaystyle \lim_{x\to -\infty} \frac{e^x}{x} = 0$

(b) $\displaystyle \lim_{t\to \infty} \frac{3t^3+2t^2+t}{1-t^3} = -3$

(c) $\displaystyle \lim_{x\to \infty} \sin x$ DNE (oscillates)

(8) Use the definition of the derivative to find $f'(2)$ (i.e. set up and evaluate a limit).

(a) $f(x) = \sqrt{x}$ $\quad \displaystyle f'(x) = \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} = \frac{1}{2\sqrt{2}}$

(b) $f(x) = x^3$ $\quad \displaystyle f'(x) = \lim_{h\to 0} \frac{(2+h)^3-2^3}{h} = 12$

(c) $f(x) = (x-1)^2$ $\quad \displaystyle f'(x) = \lim_{h\to 0} \frac{(2-1+h)^2-(2-1)^2}{h} = 2$