**Solutions to Review for Exam I**

Calculus II sec. 001 Summer 2015

Sections Covered: 4.8, 5.2, 5.3, 5.4, 5.5, 5.6, 6.1

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Evaluate the given integrals.

(a) given $\int_0^1 g(x)\,dx = 1$, and $\int_0^2 g(x)\,dx = 7$, evaluate $\int_1^2 g(x)\,dx = 6$

(b) $\int_{-1}^2 (x^2+3x-1)\,dx = \frac{9}{2}$

(c) $\int \tan^3\frac{x}{2}\sec^2\frac{x}{2}\,dx = \frac{1}{2}\tan^4\frac{x}{2}+C$

(d) $\int_0^{\frac{\pi}{6}} \frac{\sin 2x}{\cos^4 2x}\,dx = \frac{7}{6}$

(e) $\int_1^4 \frac{dy}{2\sqrt{y}(1+\sqrt{y})^2} = \frac{1}{6}$

(f) $\int \frac{x^3}{\sqrt{x^4+1}}\,dx = \frac{1}{2}\sqrt{x^4+1}+C$

(g) $\int (\sec x - \csc(2x))\,dx = \ln|\sec x+\tan x| + \frac{1}{2}\ln|\csc(2x)+\cot(2x)|+C$

(h) $\int_{-3}^{-1} \frac{x-4}{x^2}\,dx = -\ln 3 - \frac{8}{3}$

(i) $\int 3\cot w\,dw = 3\ln|\sin w|+C$

(j) $\int \left(\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+4x^2}\right)dx = \sin^{-1}x + \tan^{-1}(2x)+C$

(k) $\int \frac{2x+1}{x^2+x+2}\,dx = \ln|x^2+x+1|+C$

(2) Evaluate each integral by interpreting it in terms of areas.

(a) $\int_0^4 -\sqrt{16-x^2}\,dx = -4\pi$

(b) $\int_0^2 f(x)\,dx = \frac{5}{2}$ where $f(x) = \begin{cases} 1, & x \le 1 \\ x, & x > 1 \end{cases}$

(3) Find the area bound between the indicated curves.

(a) $x = 2y^2, \quad x = 0, \quad$ and $\quad y = 3. \quad$ Area $= 18$

(b) $y = \cos x, \quad y = \sin x, \quad$ for $\quad 0 \leq x \leq \dfrac{\pi}{2}$. Area $= 2\sqrt{2}-2$

(c) $y = 2-x^2 \quad$ and $\quad y = x, \quad$ Area $= \dfrac{9}{2}$

(4) Evaluate each derivative.

(a) $\dfrac{d}{dx}\displaystyle\int_x^{x^3} \tan t\,dt = 3x^2 \tan x^3 - \tan x$

(b) $\dfrac{d}{dx}\displaystyle\int_{\sin x}^{\cos x} \frac{dt}{t} = -\tan x - \cot x$

(5) Explain why each statement below is false.

(a) If $f$ is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x). \quad \textbf{(Note that the definite integral is a constant.)}$$

(b) If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$ for all $0 \leq x \leq 1$. **(Consider the counter example $f(x) = 1/2 - x$.)**

(c) If $f$ is continuous on $[a, b]$, then $f$ has a derivative on $[a, b]$. **(Consider the counter example $f(x) = |x|$ on $[-1, 1]$.)**

(d) If $f$ is continuous on $[a, b]$, then $\int_a^b xf(x)\,dx = \frac{x^2}{2}\int_a^b f(x)\,dx$. **(This is a pretty egregious error. Only constant *factors* may be factored out. Note that the expression on the left is a number, that on the right would be a number times $x^2$.)**

(6) A particle moves along the $x$-axis; it's acceleration $a(t)$, initial velocity $v(0)$, and initial position $s(0)$ are given by

$$a(t) = 2\cos t \quad \text{ft/s}^2, \quad v(0) = 2 \quad \text{ft/s}, \quad \text{and} \quad s(0) = 0 \quad \text{ft}.$$

Find the position $s(t)$ for all $t > 0$. $\qquad s(t) = -2\cos t + 2t + 2$

(7) If $f$ is continuous on $[a, b]$ and $m \leq f(x) \leq M$ on this interval, then $m(b-a) \leq \int_a^b f(x)\,dx \leq M(b-a)$. Use this property to show that

$$0 \leq \int_{-1}^{1/2} (1-x^2)\,dx \leq \frac{3}{2}.$$

Use the extreme value theorem, and show that for $-1 \leq x \leq \frac{1}{2}$, $\quad 0 \leq (1-x^2) \leq 1.$