

## Review for Exam I

### MATH 2306

Sections Covered: 1.1, 1.2, 2.2, 2.3, 3.1

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a)  $\frac{dy}{dt} + \frac{dx}{dt} = x^2 + y^2$

(b)  $x^3 y''' - 2x^2 y'' + 7y = \ln x$

(c)  $e^x dy = x^2 y dx$

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly or explicitly.

(a)  $\frac{d^2 y}{dx^2} + y = e^x, \quad y(x) = 2 \cos x + \frac{1}{2} e^x$

(b)  $\frac{dy}{dx} = \frac{y}{e^x} \quad e^{-x} + \ln |y| = 1$

(3) Find values of  $m$  so that the function  $y = x^m$  is a solution of the differential equation

$$x^2 y'' - 7xy' + 15y = 0$$

(4) Verify that the indicated family of functions is a solution of the given differential equation.

$$\frac{dP}{dt} = P(1-P); \quad P = \frac{c_1 e^t}{1 + c_1 e^t}$$

(5) Use the results from the previous problem to solve the I.V.P.

$$\frac{dP}{dt} = P(1-P), \quad P(0) = P_0$$

(6) Each of the first order equations is either separable or linear. Find the general solution to each equation.

(a)  $\frac{dy}{dx} = \sqrt{xy}$

(b)  $\sin x \, y' + \cos x \, y = \sec^2 x$

(c)  $\frac{dy}{dx} = y+x$

(7) Solve each IVP.

(a)  $\frac{dy}{dt} + 2ty = 4t, \quad y(0) = -1$

(b)  $e^y \ln(x) \, dx + y \, dy = 0, \quad y(1) = -1$

(c)  $\frac{du}{d\theta} + \sec \theta \, u = \cos \theta, \quad u(0) = 2$

(8) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current  $i(t)$  if  $i(0) = 0$ .

(9) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find

the amount of salt  $A(t)$  in the tank at time  $t$ .

(10) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes.