This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a) $\dfrac{dy}{dt}+\dfrac{dx}{dt}=x^2+y^2$ independent t, dependent x, y, first order, nonlinear

(b) $x^3y'''-2x^2y''+7y=\ln x$ independent x, dependent y, third order, linear

(c) $e^x dy = x^2 y dx$ independent/dependent could be either, first order, nonlinear in x, linear in y

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly or explicitly.

(a) $\dfrac{d^2y}{dx^2}+y=e^x,\quad y(x)=2\cos x+\dfrac{1}{2}e^x$ plug it in, explicit

(b) $\dfrac{dy}{dx}=\dfrac{y}{e^x},\quad e^{-x}+\ln|y|=1$ plug it in, implicit

(3) Find values of m so that the function $y=x^m$ is a solution of the differential equation

$x^2y''-7xy'+15y=0$ $m=5$ or $m=3$

(4) Solve each first order separable equation.

(a) $\dfrac{dy}{dx} = \sqrt{xy}$ $\qquad 2\sqrt{y} = \dfrac{2}{3}x^{3/2} + C$

(b) $\sin^2 x \dfrac{dy}{dx} = \sec^2 y$ $\qquad \dfrac{1}{2}y + \dfrac{1}{4}\sin(2y) = -\cot x + C$

(c) $\dfrac{dy}{dx} = \dfrac{x}{y}e^{x-y}$ $\qquad ye^y - e^y = xe^x - e^x + C$

(5) Solve each IVP.

(a) $\dfrac{dy}{dx} = \sqrt{xy}, \quad y(0) = 1$ $\qquad y = \left(\dfrac{1}{3}x^{3/2} + 1\right)^2$

(b) $e^y \ln(x)\, dx + y\, dy = 0, \quad y(1) = -1$ $\qquad e^{-y}(y+1) = x\ln x - x + 1$

(c) $y'' = -\cos x + 6x, \quad y(0) = 3, \quad y'(0) = -1$ $\qquad y = \cos x + x^3 - x + 2$

(6) Solve each IVP.

(a) $\dfrac{dy}{dx} - \tan x\, y = \sin x, \quad y(0) = 1$ $\qquad y = \dfrac{1}{2}\sin^2 x \sec x + \sec x$

(b) $x\dfrac{dy}{dx} + 3\, y = \dfrac{1}{x^2(1+x^2)}, \quad y(1) = 0$ $\qquad y = \dfrac{\tan^{-1} x}{x^3} - \dfrac{\pi}{4x^3}$

(c) $ty' + y = 2te^{2t}, \quad y(1) = 0$ $\qquad y = e^{2t} - \dfrac{e^{2t} + e^2}{2t}$

(7) Find an explicit solution to each differential equation.

(a) $y' + 3y = y^2 e^{3x}$ $\qquad y = \dfrac{e^{-3x}}{c - x}$

(c) $\dfrac{dy}{dx} + 4xy = 4x\sqrt{y}$ $\qquad y = (1 + ce^{-x^2})^2$