Review for Exam I (Solutions)

MATH 2306 sec. 58 & 59

Sections Covered: 1, 2, 3

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a) $\dfrac{dy}{dt}+\dfrac{dx}{dt}=x^2+y^2,$ independent t, dependent x, y, first orde,r nonlinear

(b) $x^3y'''-2x^2y''+7y=\ln x$ independent x, dependent y, third order, linear

(c) $e^x dy=x^2y dx$ independent/dependent could be either, first order, nonlinear in x, linear in y

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly of explicitly.

(a) $\dfrac{d^2y}{dx^2}+y=e^x, \quad y(x)=2\cos x+\dfrac{1}{2}e^x$ plug it in, explicit

(b) $\dfrac{dy}{dx}=\dfrac{y}{e^x} \quad e^{-x}+\ln|y|=1$ plug it in, implicit

(3) Find values of m so that the function $y=x^m$ is a solution of the differential equation

$x^2y''-7xy'+15y=0 \quad m=5$ or $m=3$

(4) Verify that the indicated family of functions is a solution of the given differential equation.

$\dfrac{dP}{dt}=P(1-P); \quad P=\dfrac{c_1e^t}{1+c_1e^t}$ plug it in

(5) Use the results from the previous problem to solve the I.V.P.

$$\frac{dP}{dt} = P(1-P), \quad P(0) = P_0, \quad P(t) = \frac{P_0 e^t}{1 - P_0 + P_0 e^t}$$

(6) Each of the first order equations is either separable or linear. Find the general solution to each equation.

(a) $\dfrac{dy}{dx} = \sqrt{xy} \quad 2\sqrt{y} = \dfrac{2}{3}x^{3/2}+C$

(b) $\sin^2 x \dfrac{dy}{dx} = \sec^2 y \quad \dfrac{1}{2}y+\dfrac{1}{4}\sin(2y) = -\cot x+C$

(c) $\dfrac{dy}{dx} = \dfrac{x}{y}e^{x-y} \quad ye^y-e^y = xe^x-e^x+C$

(7) Solve each IVP.

(a) $\dfrac{dy}{dx} = \sqrt{xy}, \quad y(0) = -1; \qquad y = \left(\dfrac{1}{3}x^{3/2}+1\right)^2$

(b) $e^y \ln(x)\, dx+y\, dy = 0, \quad y(1) = -1; \qquad e^{-y}(y+1) = x\ln x-x+1$ (implicitly defined)

(c) $y'' = -\cos x+6x, \quad y(0) = 3, \quad y'(0) = -1; \qquad y = \cos x+x^3-x+2$