Review for Exam I

MATH 2306

Sections Covered: 1, 2, 3, 4¹

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a)
$$\frac{dy}{dt} + \frac{dx}{dt} = x^2 + y^2$$

(b)
$$x^3y''' - 2x^2y'' + 7y = \ln x$$

(c)
$$e^x dy = x^2 y dx$$

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly or explicitly.

(a)
$$\frac{d^2y}{dx^2} + y = e^x$$
, $y(x) = 2\cos x + \frac{1}{2}e^x$

(b)
$$\frac{dy}{dx} = \frac{y}{e^x}$$
 $e^{-x} + \ln|y| = 1$

(3) Find values of m so that the function $y = x^m$ is a solution of the differential equation

$$x^2y'' - 7xy' + 15y = 0$$

¹Only first order linear is included from section 4.

(4) Solve each first order separable equation.

(a)
$$\frac{dy}{dx} = \sqrt{xy}$$

(b)
$$\sin^2 x \frac{dy}{dx} = \sec^2 y$$

(c)
$$\frac{dy}{dx} = \frac{x}{y}e^{x-y}$$

(5) Solve each IVP.

(a)
$$\frac{dy}{dx} = \sqrt{xy}$$
, $y(0) = 1$

(b)
$$e^y \ln(x) dx + y dy = 0$$
, $y(1) = -1$

(c)
$$y'' = -\cos x + 6x$$
, $y(0) = 3$, $y'(0) = -1$

(6) Solve each IVP.

(a)
$$\frac{dy}{dx}$$
 - tan $xy = \sin x$, $y(0) = 1$

(b)
$$x \frac{dy}{dx} + 3y = \frac{1}{x^2(1+x^2)}, \quad y(1) = 0$$

(c)
$$ty'+y = 2te^{2t}$$
, $y(1) = 0$