

## Review for Exam I

### MATH 2306

Sections Covered: 1, 2, 3, 4<sup>1</sup>

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

- (a)  $\frac{dy}{dt} + \frac{dx}{dt} = x^2 + y^2$     independent  $t$ , dependent  $x, y$ , first order, nonlinear
- (b)  $x^3 y''' - 2x^2 y'' + 7y = \ln x$     independent  $x$ , dependent  $y$ , third order, linear
- (c)  $e^x dy = x^2 y dx$     independent/dependent could be either, first order, nonlinear in  $x$ , linear in  $y$

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly or explicitly.

- (a)  $\frac{d^2 y}{dx^2} + y = e^x$ ,  $y(x) = 2 \cos x + \frac{1}{2} e^x$     plug it in, explicit
- (b)  $\frac{dy}{dx} = \frac{y}{e^x}$   $e^{-x} + \ln |y| = 1$     plug it in, implicit

(3) Find values of  $m$  so that the function  $y = x^m$  is a solution of the differential equation

$$x^2 y'' - 7x y' + 15y = 0 \quad m = 5 \quad \text{or} \quad m = 3$$

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<sup>1</sup>Only first order linear is included from section 4.

(4) Solve each first order separable equation.

(a)  $\frac{dy}{dx} = \sqrt{xy}$        $2\sqrt{y} = \frac{2}{3}x^{3/2} + C$

(b)  $\sin^2 x \frac{dy}{dx} = \sec^2 y$        $\frac{1}{2}y + \frac{1}{4}\sin(2y) = -\cot x + C$

(c)  $\frac{dy}{dx} = \frac{x}{y}e^{x-y}$        $ye^y - e^y = xe^x - e^x + C$

(5) Solve each IVP.

(a)  $\frac{dy}{dx} = \sqrt{xy}$ ,  $y(0) = 1$        $y = \left(\frac{1}{3}x^{3/2} + 1\right)^2$

(b)  $e^y \ln(x) dx + y dy = 0$ ,  $y(1) = -1$        $e^{-y}(y+1) = x \ln x - x + 1$

(c)  $y'' = -\cos x + 6x$ ,  $y(0) = 3$ ,  $y'(0) = -1$        $y = \cos x + x^3 - x + 2$

(6) Solve each IVP.

(a)  $\frac{dy}{dx} - \tan x y = \sin x$ ,  $y(0) = 1$        $y = \frac{1}{2}\sin^2 x \sec x + \sec x$

(b)  $x \frac{dy}{dx} + 3y = \frac{1}{x^2(1+x^2)}$ ,  $y(1) = 0$        $y = \frac{\tan^{-1} x}{x^3} - \frac{\pi}{4x^3}$

(c)  $ty' + y = 2te^{2t}$ ,  $y(1) = 0$        $y = e^{2t} - \frac{e^{2t} + e^2}{2t}$