Review for Exam I

MATH 2306

Sections Covered: 1, 2, 3, 4

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a) $\frac{dy}{dt}+\frac{dx}{dt}=x^2+y^2$ independent t, dependent x, y, first order, nonlinear

(b) $x^3y'''-2x^2y''+7y=\ln x$ independent x, dependent y, third order, linear

(c) $e^x dy=x^2y dx$ independent/dependent could be either, first order, nonlinear in x, linear in y

(To say it's *linear in y* means that when y is considered the dependent variable, the equation is linear.)

(2) Determine whether or not the given expression defines a solution to the ODE.

(a) $y(x)=2\cos x+\frac{1}{2}e^x$; $\frac{d^2y}{dx^2}+y=e^x$ It does. Use substitution to show it.

(b) $\ln(xy)=x^2+y^2$; $(x-2xy^2)\frac{dy}{dx}=(2x^2y-y)$ It does. Use implicit differentiation to show it.

(c) $y=e^x+2xe^x$; $y''-3y'+2y=0$ It does not. Plug this y into the ODE and you get $-2e^x$, not zero.

(d) $e^{-x}+\ln|y|=1$; $\frac{dy}{dx}=\frac{y}{e^x}$ It does. Use implicit differentiation to show it.

(3) Find values of m so that the function $y=x^m$ is a solution of the differential equation

(a) $x^2y''-7xy'+15y=0$ $m=5$ or $m=3$

(b) $x^2y''-xy'-2y=0$ $m=1+\sqrt{3}$ or $m=1-\sqrt{3}$

(4) Solve each first order separable equation.

(a) $\dfrac{dy}{dx} = \sqrt{xy}$ $\qquad 2\sqrt{y} = \dfrac{2}{3}x^{3/2} + C$

(b) $\sin^2 x \dfrac{dy}{dx} = \sec^2 y$ $\qquad \dfrac{1}{2}y + \dfrac{1}{4}\sin(2y) = -\cot x + C$

(c) $\dfrac{dy}{dx} = \dfrac{x}{y}e^{x-y}$ $\qquad ye^y - e^y = xe^x - e^x + C$

(5) Solve each IVP.

(a) $\dfrac{dy}{dx} = \sqrt{xy}, \quad y(0) = 1$ $\qquad y = \left(\dfrac{1}{3}x^{3/2} + 1\right)^2$

(b) $e^y \ln(x)\, dx + y\, dy = 0, \quad y(1) = -1$ $\qquad e^{-y}(y+1) = x\ln x - x + 1$

(c) $y'' = -\cos x + 6x, \quad y(0) = 3, \quad y'(0) = -1$ $\qquad y = \cos x + x^3 - x + 2$

(6) Solve each IVP.

(a) $\dfrac{dy}{dx} - \tan x\, y = \sin x, \quad y(0) = 1$ $\qquad y = \dfrac{1}{2}\sin^2 x \sec x + \sec x$

(b) $x\dfrac{dy}{dx} + 3\,y = \dfrac{1}{x^2(1+x^2)}, \quad y(1) = 0$ $\qquad y = \dfrac{\tan^{-1} x}{x^3} - \dfrac{\pi}{4x^3}$

(c) $ty' + y = 2te^{2t}, \quad y(1) = 0$ $\qquad y = e^{2t} - \dfrac{e^{2t} + e^2}{2t}$

(7) Solve each differential equation using any applicable technique

(a) $y' + 3y = y^2 e^{3x},$ $\qquad y = \dfrac{e^{-3x}}{c - x}$

(b) $(2xy^2 - 2\sin(2x))\, dx + 2x^2 y\, dy = 0$ $\qquad x^2y^2 + \cos(2x) = C$

(c) $(ye^x + y^3)\, dx + \left(2xy^2 - \dfrac{y}{1+y^2}\right) dy = 0$ $\qquad e^x + xy^2 - \tan^{-1}(y) = C$

(d) $\dfrac{dy}{dx} + 4xy = 4x\sqrt{y}$ $\qquad y = (1 + ce^{-x^2})^2$