Solutions to Review for Exam I

MATH 2306 sec. 52

Sections Covered: 1, 2, 3, 4

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.

(a) $\dfrac{dy}{dt}+\dfrac{dx}{dt}=x^2+y^2,$ independent t, dependent x, y, first order, nonlinear

(b) $x^3y'''-2x^2y''+7y=\ln x$ independent x, dependent y, third order, linear

(c) $e^x dy=x^2y dx$ independent/dependent could be either, first order, nonlinear in x, linear in y

(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly or explicitly.

(a) $\dfrac{d^2y}{dx^2}+y=e^x,\quad y(x)=2\cos x+\dfrac{1}{2}e^x,$ Just plug it in. It is explicit.

(b) $\dfrac{dy}{dx}=\dfrac{y}{e^x}\quad e^{-x}+\ln|y|=1$ Use implicit differentiation. It is implicit.

(3) (a) Find values of m so that the function $y=x^m$ is a solution of the differential equation

$x^2y''-7xy'+15y=0$ $m=3$ or $m=5$, so $y=x^3$ and $y=x^5$ are solutions.

(b) Find all values of m such that $y=e^{mx}$ is a solution of the differential equation

$y''+y'-6y=0$ $m=2$ or $m=-3$, so $y=e^{2x}$ and $y=e^{-3x}$ are solutions.

(4) (a) Verify that the indicated family of functions is a solution of the given differential equation.

$y''+y'-6y=0, \quad y=c_1e^{2x}+c_2e^{-3x}$ Just plug it in.

(b) Find the solution to the I.V.P.

$y''+y'-6y=0, \quad y(0)=1, \quad y'(0)=7$ $y=2e^{-2x}-e^{-3x}$

(5) Solve each first order separable equation.

(a) $\dfrac{dy}{dx}=\sqrt{xy}$ $2\sqrt{y}=\dfrac{2}{3}x^{3/2}+C$

(b) $\sin^2 x\dfrac{dy}{dx}=\sec^2 y$ $\dfrac{1}{2}y+\dfrac{1}{4}\sin(2y)=-\cot x+C$

(c) $\dfrac{dy}{dx}=\dfrac{x}{y}e^{x-y}$ $ye^y-e^y=xe^x-e^x+C$

(6) Find the general solution of each first order linear equation.

(a) $y'+\dfrac{1}{x}y=\dfrac{\cos x+1}{x}$ $y=\dfrac{\sin x}{x}+1+\dfrac{C}{x}$

(b) $\dfrac{dy}{dt}+2ty=te^{-t^2}$ $y=\dfrac{1}{2}te^{-t^2}+Ce^{-t^2}$

(c) $\sin(x)y'+\cos(x)y=\dfrac{1}{1+x^2}, \quad 0<x<\pi$ $y=\dfrac{\tan^{-1}x+C}{\sin x}$

(7) Solve each IVP.

(a) $\dfrac{dy}{dx}=\sqrt{xy}, \quad y(0)=1$ $2\sqrt{y}=\dfrac{2}{3}x^{3/2}+2$

(b) $\dfrac{dy}{dx}-\tan x\, y=\sin x, \quad y(0)=1$ $y=\dfrac{1}{2}\sin x\tan x+\sec x$

(c) $y''=-\cos x+6x, \quad y(0)=3, \quad y'(0)=-1$ $y=\cos x+x^3-x+2$

(d) $x\dfrac{dy}{dx}+3\,y=\dfrac{1}{x^2(1+x^2)}, \quad y(1)=0$ $y=\dfrac{\tan^{-1}x-\frac{\pi}{4}}{x^3}$