## Review for Exam I

MATH 2306
Sections Covered: 1.1, 1.2, 2.2, 2.3, 3.1

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.
(1) For each equation, specify all independent and dependent variables. Identify the given equation as Linear or Non-linear and specify the order.
(a) $\frac{d y}{d t}+\frac{d x}{d t}=x^{2}+y^{2}, \quad$ independent $t$, dependent $x, y$, first orde,r nonlinear
(b) $x^{3} y^{\prime \prime \prime}-2 x^{2} y^{\prime \prime}+7 y=\ln x \quad$ independent $x$, dependent $y$, third order, linear
(c) $\quad e^{x} d y=x^{2} y d x \quad$ independent/dependent could be either, first order, nonlinear in $x$, linear in $y$
(2) Verify that the given expression defines a solution to the ODE. State whether the solution is given implicitly of explicitly.
(a) $\frac{d^{2} y}{d x^{2}}+y=e^{x}, \quad y(x)=2 \cos x+\frac{1}{2} e^{x} \quad$ plug it in, explicit
(b) $\quad \frac{d y}{d x}=\frac{y}{e^{x}} \quad e^{-x}+\ln |y|=1 \quad$ plug it in, implicit
(3) Find values of $m$ so that the function $y=x^{m}$ is a solution of the differential equation

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x^{2} y^{\prime \prime}-7 x y^{\prime}+15 y=0 \quad m=5 \quad \text { or } \quad m=3
$$

(4) Verify that the indicated family of functions is a solution of the given differential equation.
$\frac{d P}{d t}=P(1-P) ; \quad P=\frac{c_{1} e^{t}}{1+c_{1} e^{t}} \quad$ plug it in
(5) Use the results from the previous problem to solve the I.V.P.
$\frac{d P}{d t}=P(1-P), \quad P(0)=P_{0}, \quad P(t)=\frac{P_{0} e^{t}}{1-P_{0}+P_{0} e^{t}}$
(6) Each of the first order equations is either separable or linear. Find the general solution to each equation.
(a) $\quad \frac{d y}{d x}=\sqrt{x y} \quad 2 \sqrt{y}=\frac{2}{3} x^{3 / 2}+C$
(b) $\quad \sin x y^{\prime}+\cos x y=\sec ^{2} x \quad y=\sec x+C \csc x$
(c) $\frac{d y}{d x}=y+x \quad y=-x-1+C e^{x}$
(7) Solve each IVP.
(a) $\quad \frac{d y}{d t}+2 t y=4 t, \quad y(0)=-1 \quad y=2-3 e^{-t^{2}}$
(b) $\quad e^{y} \ln (x) d x+y d y=0, \quad y(1)=-1 \quad e^{-y}(y+1)=x \ln x-x+1$
(c) $\quad \frac{d u}{d \theta}+\sec \theta u=\cos \theta, \quad u(0)=2 \quad u=\frac{\theta-\cos \theta+3}{\sec \theta+\tan \theta}$
(8) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0)=0 . \quad i(t)=50\left(1-e^{-t / 5}\right)$
(9) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of $5 \mathrm{~L} / \mathrm{min}$, and the well mixed solution is flowing out of the tank at the same rate. Find
the amount of salt $A(t)$ in the tank at time $\mathrm{t} . \quad A(t)=100-95 e^{-t / 100}$
(10) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes. $\quad A(30)=\frac{125}{32} \mathrm{lbs}$

