Practice for Exam I (Ritter) MATH 3260 Fall 2017

Sections Covered: 1.1, 1.2, 1.3, 1.4, 1.5, 1.7, 1.8, 1.9, 2.1

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Row reduce the matrix to reduced echelon form (by hand). Determine the pivot positions and pivot columns in the matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 6 & 8 & 10 \\ 6 & 8 & 10 & 4 \end{bmatrix}$$

(2) Find the solution set of the homogeneous linear system. Express the solution in parametric vector form.

$$\begin{array}{rcrcrcrcl} 2x_1 & + & 4x_2 & + & 6x_3 & + & 8x_4 & = & 0 \\ 4x_1 & + & 6x_2 & + & 8x_3 & + & 10x_4 & = & 0 \\ 6x_1 & + & 8x_2 & + & 10x_3 & + & 4x_4 & = & 0 \end{array}$$

(3) Determine the value(s) of h and k such that the system has

(a) exactly one solution,

(b) infinitely many solutions,

(c) no solutions.

$$\begin{array}{rcrcl} x_1 & + & 3x_2 & = & 2 \\ 3x_1 & + & hx_2 & = & k \end{array}$$

(4) Find the solution set for each linear system of equations. Use an augmented matrix with row operations. You may express the solution parametrically (x_1 =blah, x_2 = blah blah etc.) or in

parametric vector form, your choice.

(a)
$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & + & x_3 & = & 6 \\ x_1 & + & x_2 & - & x_3 & = & -2 \\ & & x_2 & + & 3x_3 & = & 5 \end{array}$$

(b)
$$\begin{array}{rcrcrcr} 2x_1 & + & 5x_2 & - & 3x_3 & = & 6 \\ x_1 & - & x_2 & & & = & 2 \\ 3x_1 & + & 4x_2 & - & 3x_3 & = & 8 \end{array}$$

(c)
$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 4x_3 & = & 0 \\ 2x_1 & + & 4x_2 & - & 8x_3 & = & 8 \end{array}$$

(5) Consider the given set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

(a) Determine if $\mathbf{b} = (1, 3, 1)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights. [1]

(b) Determine if $\mathbf{b} = (2, -2, -6)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights.

(c) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. If not, find a linear dependence relation.

(d) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. If not, find a linear dependence relation.

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[1] The vector $\mathbf{b}$ is equivalent to the column

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

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(e) Does the matrix equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution if $A = [\mathbf{v}_1\ \mathbf{v}_3\ \mathbf{v}_4]$? Justify your answer.

(6) Determine whether the transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ is a linear transformation. Justify your conclusion.

$$T(x_1, x_2, x_3) = (x_1 + 1, 0, 0, x_2 - x_3)$$

(7) Show that T is a linear transformation by finding a matrix that implements the mapping. (The terms x_1, x_2 etc. are entries in a vector as opposed to vectors themselves.)

$$T(x_1, x_2) = (x_1 - 2x_2, 3x_2, 0, 4x_1 + x_2)$$

(8) Show that if T is a linear transformation, then it is necessarily true that $T(\mathbf{0}) = \mathbf{0}$.

(9) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear tranformation defined by

$$T(x_1, x_2, x_3) = (x_1 - 3x_3, 2x_1 + 2x_2).$$

(a) Identify the domain and codomain.

(b) Determine if T is one to one.

(c) Determine if T is onto.

(d) Is $(1, 1)$ in the domain of T? If so, find its image. Is $(1, 1)$ in the range of T? Why or why not?

(10) Let A, B, and C be the coefficient matrices[2] for the linear systems in problem (4) (of this review), respectively. Compute each expression if it exists. If it doesn't exist, state why.

(a) $A + 2B$

(b) A^T

(c) $B + C$

(d) AC

(e) AC^T

(f) $C^T C$

[2] A is the coeff. matrix for part (a), B for part (b), etc.