Practice for Exam I (Ritter) MATH 3260 Spring 2018

Sections Covered: 1.1, 1.2, 1.3, 1.4, 1.5, 1.7

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Row reduce the matrix to reduced echelon form (by hand). Determine the pivot positions and pivot columns in the matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 6 & 8 & 10 \\ 6 & 8 & 10 & 4 \end{bmatrix}$$

(2) Find the solution set of the homogeneous linear system. Express the solution in parametric vector form.

$$\begin{array}{rcrcrcrcl} 2x_1 & + & 4x_2 & + & 6x_3 & + & 8x_4 & = & 0 \\ 4x_1 & + & 6x_2 & + & 8x_3 & + & 10x_4 & = & 0 \\ 6x_1 & + & 8x_2 & + & 10x_3 & + & 4x_4 & = & 0 \end{array}$$

(3) Determine the value(s) of h and k such that the system has

(a) exactly one solution,

(b) infinitely many solutions,

(c) no solutions.

$$\begin{array}{rcrcl} x_1 & + & 3x_2 & = & 2 \\ 3x_1 & + & hx_2 & = & k \end{array}$$

(4) Find the solution set for each linear system of equations. Use an augmented matrix with row operations. You may express the solution parametrically (x_1 =blah, $x_2 =$ blah blah etc.) or in

parametric vector form, your choice.

(a)
$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & + & x_3 & = & 6 \\ x_1 & + & x_2 & - & x_3 & = & -2 \\ & & x_2 & + & 3x_3 & = & 5 \end{array}$$

(b)
$$\begin{array}{rcrcrcr} 2x_1 & + & 5x_2 & - & 3x_3 & = & 6 \\ x_1 & - & x_2 & & & = & 2 \\ 3x_1 & + & 4x_2 & - & 3x_3 & = & 8 \end{array}$$

(c)
$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 4x_3 & = & 0 \\ 2x_1 & + & 4x_2 & - & 8x_3 & = & 8 \end{array}$$

(5) Consider the given set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

(a) Determine if $\mathbf{b} = (1, 3, 1)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights. [1]

(b) Determine if $\mathbf{b} = (2, -2, -6)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights.

(c) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. If not, find a linear dependence relation.

(d) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. If not, find a linear dependence relation.

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[1] The vector $\mathbf{b}$ is equivalent to the column

$$\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

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(e) Does the matrix equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution if $A = [\mathbf{v}_1\ \mathbf{v}_3\ \mathbf{v}_4]$? Justify your answer.

(6) Suppose the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has two solutions $\mathbf{u}$ and $\mathbf{v}$. Prove that every vector in Span$\{\mathbf{u}, \mathbf{v}\}$ is also a solution.

(7) Suppose the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent in $\mathbb{R}^5$. Show that for any vector $\mathbf{w}$ in $\mathbb{R}^5$, the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$ is linearly dependent.

(8) Find all solutions of the vector equation.

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$