Solutions to Practice for Exam I (Ritter) MATH 3260 Fall 2017

Sections Covered: 1.1, 1.2, 1.3, 1.4, 1.5, 1.7, 1.8, 1.9

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Row reduce the matrix to reduced echelon form (by hand). Determine the pivot positions and pivot columns in the matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 4 & 6 & 8 & 10 \\ 6 & 8 & 10 & 4 \end{bmatrix}$$

The row reduction can be completed in about 5 stages leading to an rref

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The pivot positions are the (1, 1), (2, 2) and (3, 4) positions, and columns 1, 2 and 4 are pivot columns.

(2) Find the solution set of the homogeneous linear system. Express the solution in parametric vector form.

$$\begin{array}{ccccccccc} 2x_1 & + & 4x_2 & + & 6x_3 & + & 8x_4 & = & 0 \\ 4x_1 & + & 6x_2 & + & 8x_3 & + & 10x_4 & = & 0 \\ 6x_1 & + & 8x_2 & + & 10x_3 & + & 4x_4 & = & 0 \end{array}$$

Reading the results off of the rref from problem (1), the solutions

$$\mathbf{x} = t \begin{bmatrix} 1 \\ -2 \\ 1 \\ 0 \end{bmatrix}, \quad -\infty < t < \infty.$$

(3) Determine the value(s) of h and k such that the system has

(a) exactly one solution, $h \neq 9$, k anything

(b) infinitely many solutions, $h = 9$, $k = 6$

(c) no solutions. $h = 9$, $k \neq 6$

$$\begin{array}{rcrcl} x_1 & + & 3x_2 & = & 2 \\ 3x_1 & + & hx_2 & = & k \end{array}$$

(4) Find the solution set for each linear system of equations. Use an augmented matrix with row operations. You may express the solution parametrically (x_1 =blah, x_2 = blah blah etc.) or in parametric vector form, your choice.

(a)
$$\begin{array}{rcrcrcr} 2x_1 & - & 2x_2 & + & x_3 & = & 6 \\ x_1 & + & x_2 & - & x_3 & = & -2 \\ & & x_2 & + & 3x_3 & = & 5 \end{array} \qquad \mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

(b)
$$\begin{array}{rcrcrcr} 2x_1 & + & 5x_2 & - & 3x_3 & = & 6 \\ x_1 & - & x_2 & & & = & 2 \\ 3x_1 & + & 4x_2 & - & 3x_3 & = & 8 \end{array} \qquad \mathbf{x} = \begin{bmatrix} \frac{16}{7} \\ \frac{2}{7} \\ 0 \end{bmatrix} + \frac{x_3}{7} \begin{bmatrix} 3 \\ 3 \\ 7 \end{bmatrix}$$

(c)
$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & - & 4x_3 & = & 0 \\ 2x_1 & + & 4x_2 & - & 8x_3 & = & 8 \end{array}$$

The system is inconsistent.

(5) Consider the given set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}.$$

(a) Determine if $\mathbf{b} = (1, 3, 1)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights. Yes, $\mathbf{b} = \mathbf{v}_1 + \frac{3}{2}\mathbf{v}_2$

(b) Determine if $\mathbf{b} = (2, -2, -6)$ is in Span$\{\mathbf{v}_1, \mathbf{v}_2\}$. If so, identify the weights. Yes, $\mathbf{b} = 2\mathbf{v}_1 - \mathbf{v}_2$

(c) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly independent. If not, find a linear dependence relation. They are linearly independent.

(d) Determine if the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent. If not, find a linear dependence relation. $6\mathbf{v}_1 + 6\mathbf{v}_2 - 4\mathbf{v}_3 - 5\mathbf{v}_4 = \mathbf{0}$ They are dependent.

(e) Does the matrix equation $A\mathbf{x} = \mathbf{0}$ have a nontrivial solution if $A = [\mathbf{v}_1\ \mathbf{v}_3\ \mathbf{v}_4]$? Justify your answer. No. If you reduce A to rref, you get I_3.

(6) Suppose the homogeneous equation $A\mathbf{x} = \mathbf{0}$ has two solutions $\mathbf{u}$ and $\mathbf{v}$. Prove that every vector in Span$\{\mathbf{u}, \mathbf{v}\}$ is also a solution.

HINT: Use properties of the matrix product to see what can be said about $\mathbf{x} = c_1\mathbf{u} + c_2\mathbf{v}$ for any choice of the scalars.

(7) Suppose the set $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is linearly dependent in $\mathbb{R}^5$. Show that for any vector $\mathbf{w}$ in $\mathbb{R}^5$, the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{w}\}$ is linearly dependent.

HINT: Use a linear dependence relation on the elements of S to find one including $\mathbf{w}$. Remember that some scalar(s) can be zero as long as at least one is not.

(8) Find all solutions of the vector equation.

$$x_1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \mathbf{x} = x_4 \begin{bmatrix} -\frac{7}{9} \\ -\frac{5}{9} \\ \frac{1}{9} \\ 1 \end{bmatrix}$$