Solutions to Review for Exam 2

MATH 1112 sections 54 Spring 2019

Sections Covered in Bittinger: 2.5, 2.4, 5.2, 5.3, 5.4, 5.5, Intro to Angles (In Miller: 2.6, 2.7, 4.2, 4.3, 4.4, 4.5, 5.1)

Calculator Policy: There will be NO calculator use on this exam. You are strongly encouraged to prepare for the exam without relying on a calculator.

There are additional pages with selected problems worked out in detail. Such solutions are available for questions marked with $\star$.

(1) Use transformations to produce a rough plot of each of the following. Label key points (such as intercepts) $\star$

(a) $y = \sqrt{x-2}$

(b) $y = \sqrt{x} - 2$

(c) $y = (x+3)^3 + 1$

(d) $y = -\sqrt{x+2}$

(2) Consider the piecewise defined function $f(x) = \begin{cases} x+2, & x < -1 \\ -x, & -1 \le x < 0 \\ 2x, & 0 \le x \end{cases}$. $\star$

Plot $y = f(x)$. Then use your graph to plot each of the following involving transformations.

(a) $y = f(x-3)$

(b) $y = -f(x)$

(c) $y = f(x) + 2$

(d) $y = f(-x)$

(3) Complete these definitions. There is a glossary linked on the course page and in D2L. Consult that to check your answer.

(a) *A function $f(x)$ is an even function if . . . for each x in the domain of f.*

(b) *A function $f(x)$ is an odd function if . . . for each x in the domain of f.*

(4) Determine algebraically whether each function is even, odd, or neither.

(a) $f(x) = x + |x|$ Neither

(b) $g(x) = \sqrt{x^2+1}$ Even

(c) $h(t) = \frac{t}{t^2+4}$ Odd

(d) $S(x) = \frac{2x-1}{(x-1)^2}$ Neither

(e) $M(x) = N(x) + N(-x)$ where N is any function whose domain is all real numbers. $\star$

(5) Identify each statement as true or false. (Full disclosure, some of these are meant to be silly.)

(a) $\dfrac{\ln(x)}{x} = \ln$ False (please recognize this as ludicrous)

(b) $\log_4(x) = \dfrac{\log_5(x)}{\log_5(4)}$ True

(c) $(e^x)^2 = e^{2x}$ True

(d) $\ln x = \dfrac{1}{x}$ False

(e) $\log_a(x-y) = \dfrac{\log_a(x)}{\log_a(y)}$ False

(f) $\log(8^9) = 9\log(8)$ True

(g) $e^{9x} = 9e^x$ False

(6) Evaluate each expression without a calculator

(a) $\log_3(1) = 0$

(b) $\log_2 \dfrac{1}{32} = -5$

(c) $\ln\sqrt{e}$ $\star$

(d) $\log(0.0001) = -4$

(e) $\log_4(2^7)$ $\star$

(f) $\log_\pi \pi = 1$

(7) Express as a single logarithm. Simplify if possible.

(a) $4\ln x + \dfrac{1}{3}\ln y - 2\ln z = \ln\left(\dfrac{x^4\sqrt[3]{y}}{z^2}\right)$

(b) $\log_2(x^3-8) - \log_2(x^2+2x+4)$ $\star$

(8) Expand as a sum or difference of logarithms.

(a) $\ln\sqrt[4]{wr^2} = \dfrac{1}{4}\ln w + \dfrac{1}{2}\ln z$

(b) $\log\sqrt[3]{\dfrac{M^2}{N}} = \dfrac{2}{3}\log M - \dfrac{1}{3}\log N$

(9) Solve each equation. Obtain an exact solution.

(a) $\log_3(x) + \log_3(x+1) = \log_3(2) + \log_3(x+3)$ $\star$

(b) $\log_3(x^2 + x) = \log_3(2) + \log_3(x + 3)$ 3 and -2

(c) $e^x + e^{-x} = 3$ $\star$

(d) $5^{x+1} = 3^{2x-1}$ $\dfrac{\ln 3 + \ln 5}{2 \ln 3 - \ln 5}$. The natural log can be replaced with any other base.

(10) Convert each angle to radian measure.

(a) $60°$ $\dfrac{\pi}{3}$

(b) $-120°$ $-\dfrac{2\pi}{3}$

(c) $18°$ $\dfrac{\pi}{10}$

(d) $-75°$ $-\dfrac{5\pi}{12}$

(11) Convert each angle to degrees.

(a) $\dfrac{\pi}{12}$ $15°$

(b) -2π $-360°$

(c) $\dfrac{4\pi}{3}$ $240°$

(d) 2 $\dfrac{360°}{\pi}$

(12) Determine each of the following.

(a) The arclength of a circle of radius 5 ft subtended by a central angle of $120°$. $\dfrac{10\pi}{3}$ ft

(b) The area of a sector of a circle of radius 5 ft for which the central angle is $120°$. $\dfrac{25\pi}{3}$ ft^2

(c) The distance traveled by a point on a minute hand of a clock between 1:45 pm and 2:05 pm if the minute hand is 6 inches long.$\star$

(13) Determine if the given angles are complements, supplements, coterminal, or none of these three things.

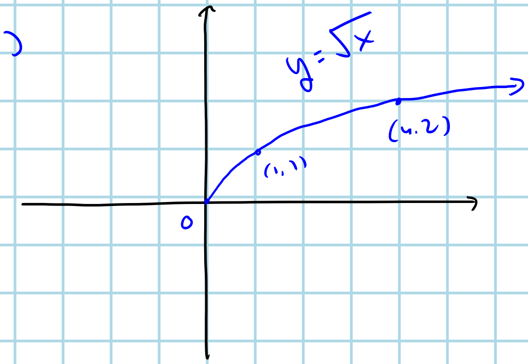
(a) $\dfrac{\pi}{3}$ and $\dfrac{\pi}{6}$ Complementary

(b) $\dfrac{4\pi}{3}$ and $-\dfrac{2\pi}{3}$ Co-terminal

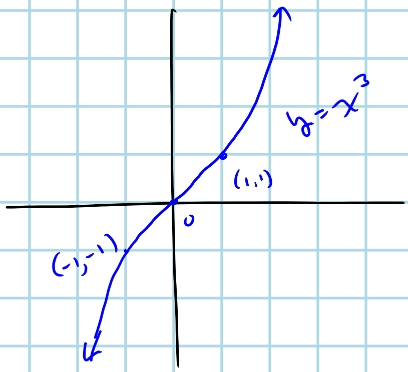
(c) $137°$ and $43°$ Supplementary

(d) $\dfrac{\pi}{2}$ and $-270°$ Co-terminal

1)

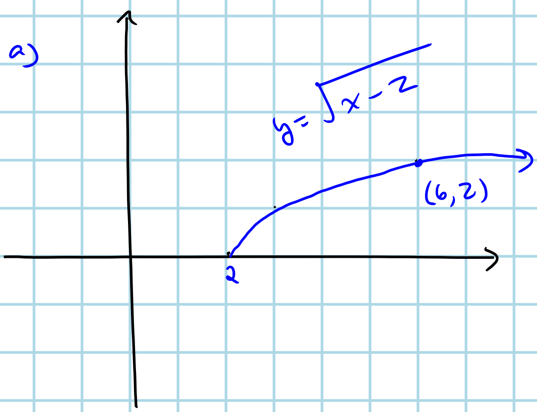


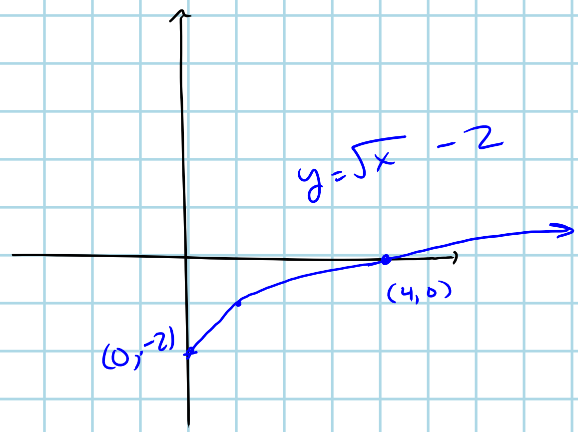
This is the parent function for a) b) and d)



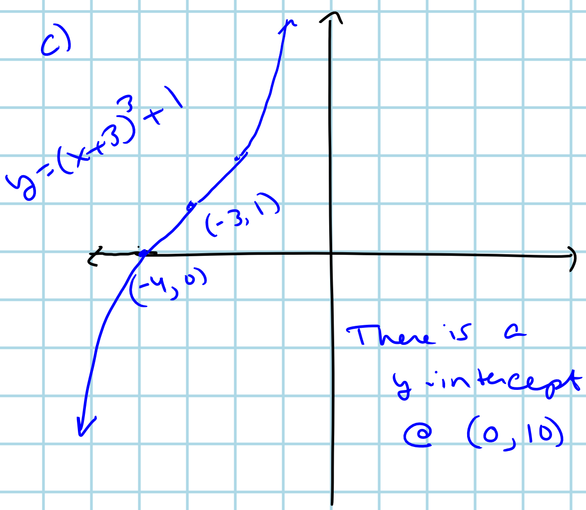
This is the parent function for c)

a)



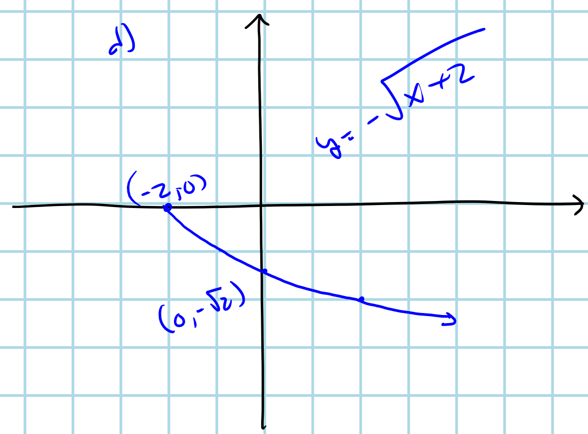


c)



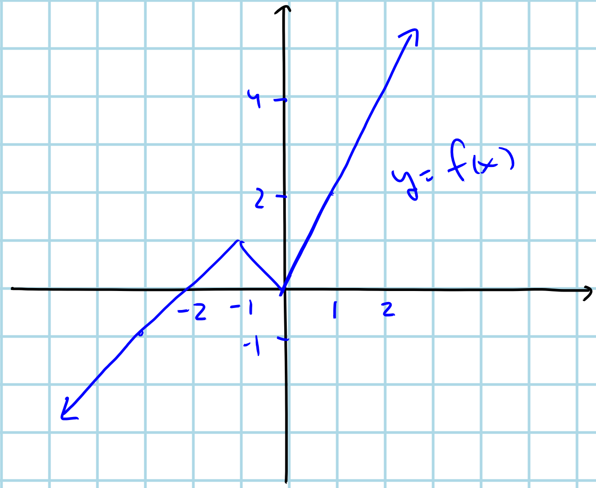
There is a y intercept @ (0,10)

d)

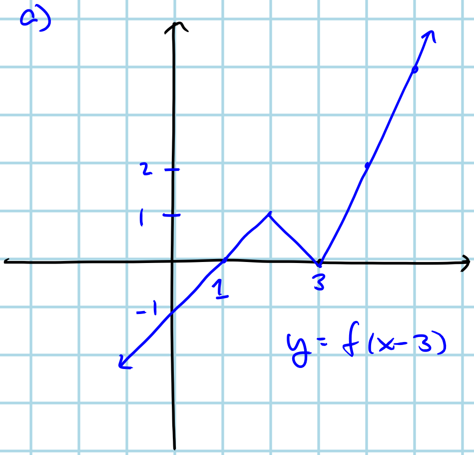


a)

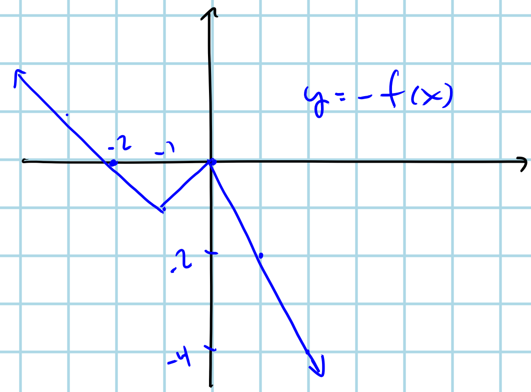
$$f(x) = \begin{cases} x+2, & x < -1 \\ -x, & -1 \le x < 0 \\ 2x, & 0 \le x \end{cases}$$



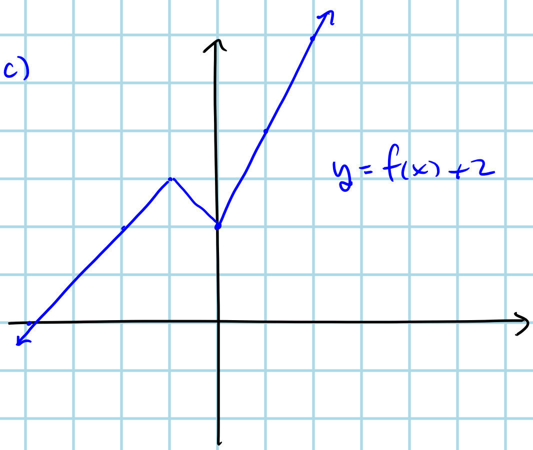
a)



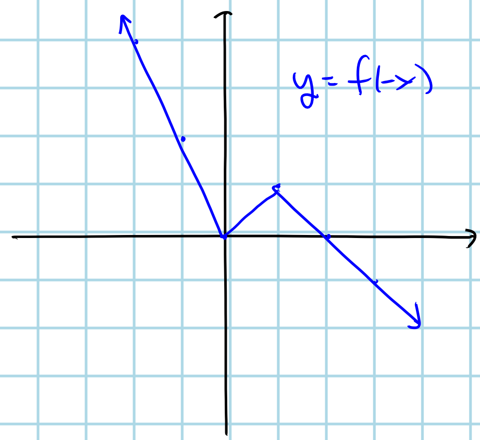
b)



c)



d)



4 e) $M(x) = N(x) + N(-x)$ This is an even function

Note that $M(-x) = N(-x) + N(-(-x))$ evaluate @ $-x$

$= N(-x) + N(x)$ $\quad -(-x) = x$

$= N(x) + N(-x)$ $\quad a+b = b+a$

$= M(x)$

That is, $M(-x) = M(x)$. So M is even.

6) c) $\ln\sqrt{e} = \ln e^{1/2} = \frac{1}{2}\ln e = \frac{1}{2}(1) = \frac{1}{2}$ $\quad \ln e = 1$

e) $\log_4(2^7) = 7\log_4(2) = 7\left(\frac{1}{2}\right) = \frac{7}{2}$ $\quad 2 = 4^{1/2}$

7 b) $\log_2(x^3-8) - \log_2(x^2+2x+4) = \log_2\left(\frac{x^3-8}{x^2+2x+4}\right)$

Difference of cubes

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

so $x^3 - 8 = (x-2)(x^2+2x+2^2)$

$= \log_2\left(\frac{(x-2)(x^2+2x+4)}{x^2+2x+4}\right)$

$= \log_2(x-2)$

9 a) $\log_3(x) + \log_3(x+1) = \log_3(2) + \log_3(x+3)$

$\log_3(x(x+1)) = \log_3(2(x+3))$ $\quad \log_a(MN) = \log_a M + \log_a N$

$x(x+1) = 2(x+3)$ $\quad \log_a x = \log_a y \Rightarrow x = y$

$x^2 + x = 2x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ $\quad x = 3$ or $x = -2$

These solutions have to be verified in the original equation

when $x=3$ $\quad \log_3(3)+\log_3(3+1) \stackrel{?}{=} \log_3(2)+\log_3(3+3)$

$$\log_3(3\cdot 4) \stackrel{?}{=} \log_3(2\cdot 6)$$

yes they both equal $\log_3(12)$

when $x=-2$ $\quad \log_3(-2)$ is not defined

-2 is <u>not</u> a solution

The only solution is 3.

9 c) $e^x+e^{-x}=3$

$$e^x(e^x+e^{-x}) = e^x(3)$$

multiply by e^x

$$(e^x)^2+e^x e^{-x} = 3e^x$$

$e^x\cdot e^{-x}=1$

$$(e^x)^2+1 = 3e^x$$

$$(e^x)^2-3e^x+1=0$$

Let $u=e^x$ $\quad u^2-3u+1=0$ $\quad u=\dfrac{3\pm\sqrt{3^2-4\cdot1\cdot1}}{2}=\dfrac{3\pm\sqrt{5}}{2}$

Both $\dfrac{3+\sqrt{5}}{2}$ and $\dfrac{3-\sqrt{5}}{2}$ are positive, so we can take

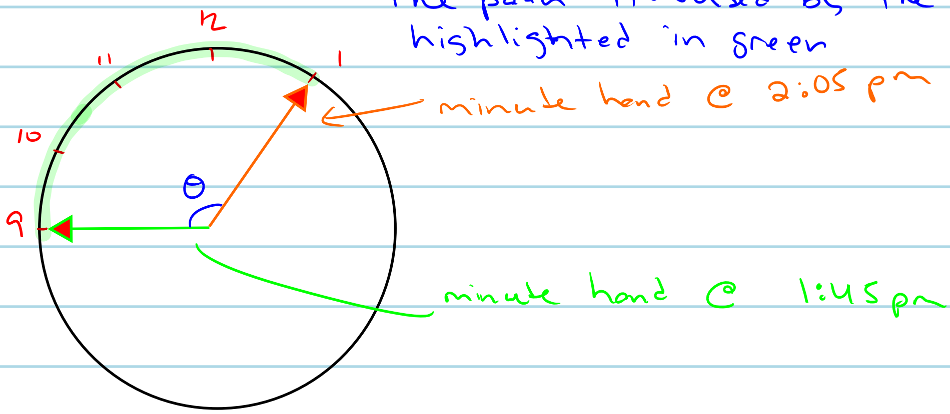
the natural log of both.

$u=e^x$ so $x=\ln u$

There are two solutions

$$x=\ln\left(\frac{3+\sqrt{5}}{2}\right) \quad \text{or} \quad x=\ln\left(\frac{3-\sqrt{5}}{2}\right)$$

The path traversed by the tip is highlighted in green



The hand passes through an angle $\theta = \frac{4}{12}(2\pi) = \frac{2\pi}{3}$

Note that that is 4 out of 12 tick marks on the clock face (9 to 10 to 11 to 12 to 1)

Arc length $s = r\theta$ $r = 6$ in and $\theta = \frac{2\pi}{3}$

so the distance

$$s = (6\text{ in})\left(\frac{2\pi}{3}\right) = 4\pi \text{ in}$$