Sections Covered: 2.4, JIT 25, 26 & 27, 3.2, JIT 15 & 16, 3.3, 4.3, 4.1, 4.2, JIT 22, 23, & 24, 4.5

Calculator Policy: You will be allowed to use a calculator as long as it does not have symbolic capabilities. However, there is not one problem on this review that actually requires a calculator.

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Factor each polynomial completely using the techniques of factoring by grouping, factoring quadratics using the ac method, and known special factorizations such as recognizing perfect squares, differences of squares, differences of cubes and sums of cubes.

  (a) $2x^3 + 6x^2 - 4x - 12$

  (b) $x^4 - 20x^2 + 64$

  (c) $q^4 + q^3 - q - 1$

  (d) $6x^2 + 23x + 20$

  (e) $8t^3 + y^6$

  (f) $2a^4 + 9a^2 - 5$

(2) Find all solutions of the polynomial equation (Hint: Factor completely first. If you think you can *see* all of the answers without work, you are mistaken.)

(a) $2(x+1)(x-2)^2 + 2(x+1)^2(x-2) = 0$

(b) $6(2x-1)^2(x+4)^5 + 5(2x-1)^3(x+4)^4 = 0$

(c) $2(x-3)(x-2)^3 + 3(x-3)^2(x-2)^2 = 0$

(3) All of the following equations are FALSE. Sadly, each appears as a common mistake arising from algebraic manipulations that aren't legitimate. Identify the error in each statement. If possible, determine what a correct statement would be. For example:

$$\frac{1}{x} + \frac{x^2}{3} = \frac{1+x^2}{x+3}$$ Addition of fractions requires a common denominator.

A correct version of this could be $\frac{1}{x} + \frac{x^2}{3} = \frac{3+x^2}{3x}$.

(a) $(x+3)^2 = x^2 + 9$

(b) $\frac{x^2+x+3}{x+4} = \frac{x^2+3}{4}$

(c) $\sqrt{x^2+1} = x+1$

(d) If $f(x) = x^3$ then $f(x+2) = x^3 + 2$

(e) If $(x-2)(x+3) = 5$, then $x - 2 = 5$ or $x + 3 = 5$

(4) Determine algebraically whether each function is even, odd, or neither.

(a) $f(x) = x + |x|$

(b) $g(x) = \sqrt{x^2 + 1}$

(c) $h(t) = \frac{t}{t^2+4}$

(d) $S(x) = \frac{2x-1}{(x-1)^2}$

(e) $M(x) = N(x) + N(-x)$ where N is any function whose domain is all real numbers.

(5) Evaluate or simplify each expression.

(a) $32^{1/5}$

(b) $\sqrt[3]{8x^6}$

(c) $\dfrac{\sqrt{2x^2}}{\sqrt{32x^4}}$ assume $x > 0$

(d) $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{5})$

(6) Write each quadratic function in vertex form. Determine the location of the vertex and the equation of the axis of symmetry.

(a) $g(x) = 2x^2 - 4x + 5$

(b) $P(x) = x^2 - 6x + 18$

(c) $f(x) = -2x^2 + 16x - 27$

(7) Perform the indicated operations and simplify the result.

(a) $\dfrac{4}{6x^2} - \dfrac{1}{3x^5} + \dfrac{5}{2x^3}$

(b) $\dfrac{4}{y+2} - \dfrac{1}{y} + 1$

(c) $\dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

(d) $\dfrac{\frac{x}{x-2} - 2}{x - 4}$

(8) For each rational function, identify each of the following:

   i The domain,

   ii the equation of all vertical asymptotes (if any),

   iii the equation of any horizontal asymptote,

   iv all points at which the graph crosses a horizontal asymptote (if it does).

(a) $f(x) = \dfrac{-2x^2 + x - 1}{x^2 - 1}$

(b) $g(x) = \dfrac{x^2 - x - 12}{x - 4}$

(c) $H(x) = \dfrac{x + 1}{x^2 + 3}$

(d) $R(x) = \dfrac{x^2 + x - 6}{3x^2 - 21x + 30}$

(9) Perform the indicated division to find the quotient $Q(x)$ and remainder $R(x)$ given the polynomial $P(x)$ and divisor $d(x)$ (that is, divide P by d).

(a) $(x^3 + 5x^2 + 6x - 4) \div (x + 3)$

(b) $\dfrac{x^2}{x^2 - 4}$

(10) An open box is to be constructed from a single piece of cardboard that is 10 inches by 12 inches by cutting out squares from the corners and turning up the edges (see the diagram). Let x be the edge length of the cut-out squares.
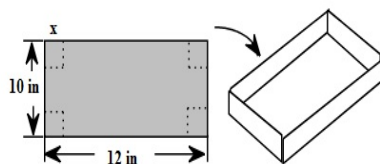
(a) Find a formula for the volume $V(x)$ as a polynomial function of x. What is it's domain?

(b) If the area of the base must be 60 sq. in., find the value of x.

(11) (a) Two real numbers sum to 5. What is the maximum value of their product? (Hint: Set up a quadratic function representing the product, and remember that a parabola has a maximum or minimum value at its vertex.)

(b) Try this problem again replacing 5 with any positive number A (i.e. if two real numbers sum to A, find the maximum value of their product).