Solutions to Review for Exam II

MATH 1113 sections 51 & 52 Fall 2018

Sections Covered:

Calculator Policy: There will be NO calculator use on this exam. You are strongly encouraged to prepare for the exam without relying on a calculator.

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Factor each polynomial completely using the techniques of factoring by grouping, factoring quadratics using the ac method, and known special factorizations such as recognizing perfect squares, differences of squares, differences of cubes and sums of cubes.

(a) $2x^3+6x^2-4x-12 \quad = 2(x-\sqrt{2})(x+\sqrt{2})(x+3)$

(b) $x^4-20x^2+64 \quad = (x-4)(x+4)(x-2)(x+2)$

(c) $q^4+q^3-q-1 \quad = (q-1)(q^2+q+1)(q+1)$

(d) $6x^2+23x+20 \quad = (2x+5)(3x+4)$

(e) $8t^3+y^6 \quad = (2t+y^2)(4t^2-2ty^2+y^4)$

(f) $2a^4+9a^2-5 \quad = (\sqrt{2}a-1)(\sqrt{2}a+1)(a^2+5)$

(2) Find all solutions of the polynomial equation (Hint: Factor completely first. If you think you can *see* the answers without work, you are mistaken.)

(a) $2(x+1)(x-2)^2+2(x+1)^2(x-2)=0 \quad x=-1, \quad x=\frac{1}{2}, \quad x=2$

(b) $6(2x-1)^2(x+4)^5+5(2x-1)^3(x+4)^4=0 \quad x=-4, \quad x=-\frac{19}{16}, \quad x=\frac{1}{2}$

(c) $2(x-3)(x-2)^3+3(x-3)^2(x-2)^2=0 \quad x=2, \quad x=\frac{13}{5}, \quad x=3$

(3) All of the following equations are FALSE. Sadly, each appears as a common mistake arising from algebraic manipulations that aren't legitimate. Identify the error in each statement. If possible, determine what a correct statement would be. For example:

$\frac{1}{x}+\frac{x^2}{3}=\frac{1+x^2}{x+3}$ Addition of fractions requires a common denominator. A correct version of this could be $\frac{1}{x}+\frac{x^2}{3}=\frac{3+x^3}{3x}$.

(a) $(x+3)^2 = x^2 + 9$ Powers don't distribute over sums. $(x+3)^2 = x^2 + 6x + 9$

(b) $\frac{x^2+x+3}{x+4} = \frac{x^2+3}{4}$ Only like FACTORs cancel in a ratio; terms in sums do not.

(c) $\sqrt{x^2+1} = x + 1$ Again, powers don't distribute over a sum.

(d) If $f(x) = x^3$ then $f(x+2) = x^3 + 2$ $f(x+2)$ is f evaluated at $x+2$ as opposed to 2 added to $f(x)$. For this f, $f(x+2) = (x+2)^3 = x^3 + 6x^2 + 12x + 8$.

(e) If $(x-2)(x+3) = 5$, then $x-2 = 5$ or $x+3 = 5$ The zero product property only applies to a zero product. The correct approach to this would be to write $(x-2)(x+3) - 5 = 0$, then factor the left side.

(4) Determine algebraically whether each function is even, odd, or neither.

(a) $f(x) = x + |x|$ Neither

(b) $g(x) = \sqrt{x^2+1}$ Even

(c) $h(t) = \frac{t}{t^2+4}$ Odd

(d) $S(x) = \frac{2x-1}{(x-1)^2}$ Neither

(e) $M(x) = N(x) + N(-x)$ where N is any function whose domain is all real numbers. This is even. Note that $M(-x) = N(-x) + N(-(-x)) = N(-x) + N(x) = M(x)$. It doesn't matter what function N is!

(5) Evaluate or simplify each expression.

(a) $32^{1/5}$ $= 2$

(b) $\sqrt[3]{8x^6}$ $= 2x^2$

(c) $\dfrac{\sqrt{2x^2}}{\sqrt{32x^4}}$ $= \frac{1}{4x}$ assume $x > 0$

(d) $(\sqrt{3} - \sqrt{5})(\sqrt{3} + \sqrt{5})$ $= -2$

(6) Write each quadratic function in vertex form. Determine the location of the vertex and the equation of the axis of symmetry.

(a) $g(x) = 2x^2 - 4x + 5$ $g(x) = 2(x-1)^2 + 3$, vertex at $(1, 3)$, axis of symmetry $x = 1$

(b) $P(x) = x^2 - 6x + 18$ $P(x) = (x-3)^2 + 9$, vertex at $(3, 9)$, axis of symmetry $x = 3$

(c) $f(x) = -2x^2 + 16x - 27$ $g(x) = -2(x-4)^2 + 5$, vertex at $(4, 5)$, axis of symmetry $x = 4$

(7) Perform the indicated operations and simplify the result.

(a) $\dfrac{4}{6x^2} - \dfrac{1}{3x^5} + \dfrac{5}{2x^3} \quad = \dfrac{4x^3 + 15x^2 - 2}{6x^5}$

(b) $\dfrac{4}{y+2} - \dfrac{1}{y} + 1 \quad = \dfrac{y^2 + 5y - 2}{y(y+2)}$

(c) $\dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} \quad = \dfrac{-2x - h}{x^2(x+h)^2}$ (for $h \neq 0$)

(d) $\dfrac{\frac{x}{x-2} - 2}{x - 4} \quad = -\dfrac{1}{x-2}$ (for $x \neq 4$)

(8) For each rational function, identify each of the following:

i The domain,

ii the equation of all vertical asymptotes (if any),

iii the equation of any horizontal asymptote,

iv all points at which the graph crosses a horizontal asymptote (if it does).

(a) $f(x) = \dfrac{-2x^2 + x - 1}{x^2 - 1}$ (b) $g(x) = \dfrac{x^2 - x - 12}{x - 4}$

(c) $H(x) = \dfrac{x + 1}{x^2 + 3}$ (d) $R(x) = \dfrac{x^2 + x - 6}{3x^2 - 21x + 30}$

(a) i. $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$, ii. $x = -1$ and $x = 1$, iii. $y = -2$, iv. It crosses at $(3, -2)$.

(b) i. $(-\infty, 4) \cup (4, \infty)$, ii. There are none. , iii. There is no such asymptote., iv. There is no asymptote to cross.

(c) i. $(-\infty, \infty)$, ii. There are none., iii. $y = 0$, iv. It crosses at $(-1, 0)$.

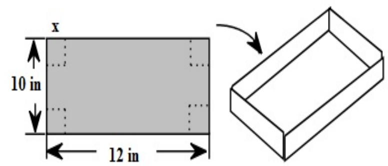
(a) i. $(-\infty, 2) \cup (2, 5) \cup (5, \infty)$, ii. $x = 5$, iii. $y = \frac{1}{3}$, iv. It does not cross. (The graph does have a hole in it at $\left(2, \frac{1}{3}\right)$).

(9) Perform the indicated division to find the quotient $Q(x)$ and remainder $R(x)$ given the polynomial $P(x)$ and divisor $d(x)$ (that is, divide P by d).

(a) $(x^3 + 5x^2 + 6x - 4) \div (x + 3)$ $\quad x^3 + 5x^2 + 6x - 4 = (x^2 + 2x)(x + 3) - 4$

(b) $\dfrac{x^2}{x^2 - 4} \quad \dfrac{x^2}{x^2 - 4} = 1 + \dfrac{4}{x^2 - 4}$

(10) An open box is to be constructed from a single piece of cardboard that is 10 inches by 12 inches by cutting out squares from the corners and turning up the edges (see the diagram). Let x be the edge length of the cut-out squares.



(a) Find a formula for the volume $V(x)$ as a polynomial function of x. What is it's domain? $V(x) = 4x^3 - 44x^2 + 120x$, $0 < x < 5$ (Note that the dimensions of the box are $x" \times (12 - 2x)" \times (10 - 2x)"$ and all side lengths must be positive.)

(b) If the area of the base must be 60 sq. in., find the value of x. $x = \dfrac{11 - \sqrt{61}}{2}$ (Solve the quadratic equation $4x^2 - 44x + 120 = 60$ subject to the condition that $x < 5$.)

(11) (a) Two real numbers sum to 5. What is the maximum value of their product? (Hint: Set up a quadratic function representing the product, and remember that a parabola has a maximum or minimum value at its vertex.) The maximum product is $\dfrac{25}{4}$. (This is the y value of the vertex of $y = 5x - x^2$.)

(b) Try this problem again replacing 5 with any positive number A (i.e. if two real numbers sum to A, find the maximum value of their product). The maximum product is $\dfrac{A^2}{4}$, the y value of the vertex of $y = Ax - x^2$.