Review for Exam II

MATH 1190

Sections Covered: 2.2, 2.3, 2.4, 2.5, 3.1, 3.2, 3.3

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Use any appropriate derivative rules to find the derivative of the indicated function.

(a) $y = \sqrt{x^3+1}$ $\quad y' = \dfrac{3x^2}{2\sqrt{x^3+1}}$

(b) $f(x) = \cot(e^{2x})$ $\quad f'(x) = -2e^{2x}\csc^2(e^{2x})$

(c) $g(t) = t^2\sin(t)$ $\quad g'(t) = 2t\sin(t) + t^2\cos(t)$

(d) $y = \sec^3(x)$ $\quad \dfrac{dy}{dx} = 3\sec^3(x)\tan(x)$

(e) $y = \ln(x^3+2x+1)$ $\quad \dfrac{dy}{dx} = \dfrac{3x^2+2}{x^3+2x+1}$

(f) $f(x) = \dfrac{x}{\sin x + 1}$ $\quad f'(x) = \dfrac{\sin x + 1 - x\cos x}{(\sin x + 1)^2}$

(g) $h(x) = \dfrac{\ln x}{x}$ $\quad h'(x) = \dfrac{1-\ln x}{x^2}$

(h) $y = \sqrt[4]{x^5}$ $\quad y' = \dfrac{5}{4}\sqrt[4]{x}$

(i) $f(x) = \sin^{-1}(x+1)$ $\quad f'(x) = \dfrac{1}{\sqrt{1-(x+1)^2}}$

(j) $h(t) = (\tan^{-1} t)^2$ $\quad h'(t) = \dfrac{2\tan^{-1} t}{1+t^2}$

(k) $y = \dfrac{x^3-1}{\sqrt{x}}$ $\quad \dfrac{dy}{dx} = \dfrac{5}{2}x^{3/2} + \dfrac{1}{2}x^{-3/2}$

(2) A particle moves along the x-axis so that it's position at time t is given by $s(t) = \frac{t^3}{6} - t^2 - 6t$.

(a) Find the velocity $v(t)$. $v(t) = \frac{t^2}{2} - 2t - 6$

(b) Determine the positive time value at which the velocity is zero. $t = 6$

(c) Determine the acceleration $a(t)$. $a(t) = t - 2$

(d) Determine the positive time value at which the acceleration is zero. $t = 2$

(3) Determine the x-values of all points on the graph of f at which the tangent line is horizontal.

(a) $f(x) = (x{+}1)^7(2x{-}3)^4$ At $x = -1$, $x = \frac{3}{2}$, and $x = \frac{13}{22}$

(b) $f(x) = \sin(2x)$, $0 \le x \le 2\pi$ At $x = \frac{\pi}{4}$, $x = \frac{3\pi}{4}$, $x = \frac{5\pi}{4}$, and $x = \frac{7\pi}{4}$

(c) $f(x) = xe^x$, At $x = -1$.

(4) Find the equation of the line tangent to the graph of the function at the indicated point.

(a) $f(x) = xe^x$ at $(1, e)$ $y = 2ex{-}e$

(b) $f(x) = \dfrac{\cos x}{1 + \sin x}$ at $(0, 1)$ $y = -x{+}1$

(5) Find the first, second and third derivative of

(a) $y = xe^x$ $\dfrac{dy}{dx} = (x{+}1)e^x$, $\dfrac{d^2y}{dx^2} = (x{+}2)e^x$, $\dfrac{d^3y}{dx^3} = (x{+}3)e^x$

(b) $f(x) = \tan x$ $f'(x) = \sec^2 x$, $f''(x) = 2\sec^2 x \tan x$, $f'''(x) = 4\sec^2 x \tan^2 x{+}2\sec^4 x$

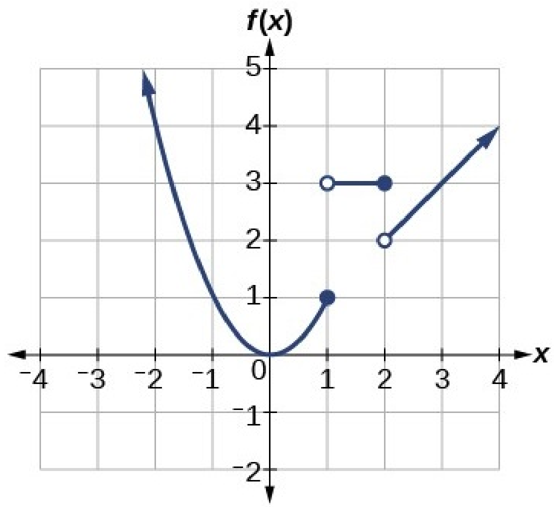
(6) Find the indicated derivative.

(a) $x^3y^2 = e^x{+}e^y$, $\dfrac{dy}{dx} = \dfrac{e^x - 3x^2y^2}{2x^3y - e^y}$

(b) $y\tan(y) = x\sin(x)$, $\dfrac{dy}{dx} = \dfrac{\sin(x) + x\cos(x)}{\tan(y) + y\sec^2(y)}$

(c) $y = (\cos x)^x$, $\dfrac{dy}{dx} = (\cos x)^x(\ln(\cos x){-}x\tan x)$

(7) Refer to the figure showing $y = f(x)$.



(a) Identify the points on the graph at which f is not differentiable. At the discontinuities at 1 and 2.

(b) Evaluate $f'(0) = 0$ (horizontal tangent)

(c) Identify any intervals over which $f'(x)$ is constant. $(1, 2)$ and $(2, \infty)$ (assuming the behaviour remains as shown)

(d) Evaluate $f'(3) = 1$

(e) Is $f'(-1)$ positive, negative, or zero? It's negative; a tangent line would be going down.

(8) Suppose f and g are differentiable functions and we know that

$$f(2) = 3, \quad f(0) = 4, \quad f(1) = 3, \quad f'(2) = 7, \quad f'(0) = 3, \quad f'(1) = 2$$

$$g(0) = 2, \quad g(4) = -2, \quad g(1) = 6, \quad g'(0) = -1, \quad g'(4) = 2, \quad g'(1) = -3$$

Evaluate if possible.

(a) If $y = f(x)g(x)$, $\quad y'(1) = f'(1)g(1) + f(1)g'(1) = 3$

(b) if $z = \frac{f(x)}{g(x)}$, $\quad z'(0) = \frac{f'(0)g(0) - f(0)g'(0)}{(g(0))^2} = \frac{5}{2}$

(c) If $h = f \circ g$, $\quad h'(0) = f'(g(0))g'(0) = f'(2)g'(0) = -7$

(d) If $s = g \circ f$, $\quad s'(0) = g'(f(0))f'(0) = g'(4)f'(0) = 6$

(e) If $h = f \circ g$, $\quad h'(2) = f'(g(2))g'(2)$, but there is not enough information