Review for Exam II

Calculus II sec. 001 Summer 2015

Sections Covered:6.2, 6.3, 6.4, 6.5, 6.6, 7.1, 7.2

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) The region in the first quadrant bounded by the curves $y = x^2$, $y = 2 - x^2$ and the y-axis is

(a) rotated about the x-axis. Find the volume of the resulting solid. Ans: $\frac{8\pi}{3}$

(b) rotated about the y-axis. Find the volume of the resulting solid. Ans: π

(2) A solid has as its base the same first quadrant region from problem (1). Cross sections taken perpendicular to the x-axis are

(a) squares with one side in the xy-plane. Find the volume of the solid. Ans: $\frac{32}{15}$

(b) semi-circles with diameter in the xy-plane. Find the volume of the solid. Ans: $\frac{4\pi}{15}$

(3) The region bounded by the x-axis, the y-axis and the curve $y = \cos x$ for $0 \le x \le \frac{\pi}{2}$ is rotated about the y-axis to generate a solid. Use the method of shells to find its volume. Ans: $\pi^2 - 2\pi$

(4) A 3 lb force is required to compress a spring 6 inches from its equilibrium length. Find the work done compressing this spring from equilibrium length to 1 foot beyond equilibrium. $k = 6$ lb/ft, and $W = 3$ ft lb

(5) A 60 lb chain is 20 feet long and has a uniform density. The chain hangs over a bridge and is pulled up by a winch. Find the work done lifting the chain. $W = 600$ ft lb

(6) Find an integral representation for the length of the curve $y = e^{\frac{x}{2}}$ from $x = 1$ to $x = 4$. Do not evaluate the integral.

$$s = \int_1^4 \sqrt{1 + \frac{1}{4}e^x}\, dx$$

(7) Find the length of the curve $y = \frac{1}{3}(x^2 + 2)^{3/2}$ from $x = 2$ to $x = 4$. $s = \frac{62}{3}$

(8) Evaluate each integral using any applicable method.

(a) $\displaystyle\int x\sec^2 x\,dx \quad = x\tan x - \ln|\sec x| + C$

(b) $\displaystyle\int 2xe^{x^2}\,dx \quad = e^{x^2} + C$

(c) $\displaystyle\int 2xe^{x}\,dx \quad = 2xe^{x} - 2e^{x} + C$

(d) $\displaystyle\int \sin^2\theta\,d\theta \quad = \frac{\theta}{2} - \frac{1}{4}\sin(2\theta) + C$

(e) $\displaystyle\int \tan^{-1} t\,dt \quad = x\tan^{-1} x - \frac{1}{2}\ln|1+x^2| + C$

(f) $\displaystyle\int \sec^4 x\tan x\,dx \quad = \frac{\sec^4 x}{4} + C \quad = \quad \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + K$

(g) $\displaystyle\int \cos^3 t\sin^2 t\,dt \quad = \frac{\sin^3 t}{3} - \frac{\sin^5 t}{5} + C$

(h) $\displaystyle\int \sqrt{\cot x}\,\csc^2 x\,dx \quad = -\frac{2}{3}(\cot x)^{3/2} + C$

(9) Evaluate the integral by first using a substitution and then integration by parts.

$$\int e^{\sqrt{x}}\,dx \quad = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$