

## Review for Exam 2

### MATH 2306

Sections Covered: 4 (exact), 5, 6, 7

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) Determine whether the equation is exact. If not exact, determine if there is an integrating factor depending only on  $x$  or only on  $y$ . If the equation is exact, or can be made exact via an integrating factor, find the general solution.

(a)  $(2xy^2 - 2\sin(2x)) dx + 2x^2y dy = 0$       $x^2y^2 + \cos(2x) = C$

(b)  $(ye^x + y^3) dx + \left(2xy^2 - \frac{y}{1+y^2}\right) dy = 0$       $e^x + xy^2 - \tan^{-1}(y) = C$

(c)  $(2y^3e^{2y}) dx + (3xy^2e^{2y} + 2xy^3e^{2y}) dy = 0$       $x^2y^3e^{2y} = C$

(2) Determine whether the indicated set of functions forms a fundamental solution set for the given ODE.

(a)  $y_1 = x$ ,  $y_2 = \frac{1}{x}$       $x^2y'' + xy' - y = 0$ ,  $x > 0$      **yes**

(b)  $y_1 = xe^x$ ,  $y_2 = e^{2x}$       $y'' - 2y' + y = 0$ ,     **no,  $y_2$  doesn't solve it**

(c)  $y_1 = e^{2x}$ ,  $y_2 = e^{2x+1}$       $y'' + y' - 6y = 0$ ,     **no, they're linearly dependent**

(d)  $y_1 = e^{2x}$ ,  $y_2 = e^{-3x}$ ,  $y_3 = 1$       $y''' + y'' - 6y' = 0$ ,     **yes**

(3) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current  $i(t)$  if  $i(0) = 0$ .      $i(t) = 50(1 - e^{-t/5})$

(4) An RC series circuit with resistance of 10 ohms and capacitance of 0.1 farads has electromotive force of  $E(t) = 20te^{-t}$  applied to it. Find the charge on the capacitor  $q(t)$  if  $q(0) = 0$ .  $q(t) = t^2e^{-t}$

(5) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find the amount of salt  $A(t)$  in the tank at time  $t$ .  $A(t) = 100 - 95e^{-t/100}$

(6) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes.  $A(30) = \frac{125}{32}$  lbs

(7) A population of bacteria experience exponential growth. If the initial population  $P(0) = 1000$ , and the population doubles every 4 hours, determine the population  $P(t)$  for all  $t > 0$ .  
 $P(t) = 1000e^{\left(\frac{\ln 2}{4}\right)t}$  for  $t$  in hours

(8) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a)  $(x-1)y'' - xy' + y = 0 \quad x > 1, \quad y_1(x) = e^x \quad y_2(x) = x$

(b)  $x^2y'' + 3xy' - 3y = 0 \quad x > 0, \quad y_1(x) = x \quad y_2(x) = x^{-3}$

(9) Solve the IVP. One solution to the differential equation is provided. (No need to repeat the work already done!)

$$(x-1)y'' - xy' + y = 0 \quad x > 1, \quad y_1(x) = e^x, \quad y(2) = 0, \quad y'(2) = 1 \quad y = 2e^{-2}e^x - x$$