Solutions to Review for Exam 2

MATH 2306

Sections Covered: 4, 5, 6, 7

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Determine whether the set of functions is linearly dependent or linearly independent on the indicated interval.

(a) $y_1(x) = e^{x+1}, \quad y_2(x) = e^{x-1}, \quad (-\infty, \infty)$ Dependent

(b) $f_1(x) = \sin x, \quad f_2(x) = \tan x, \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ Independent

(c) $g_1(t) = t, \quad g_2(t) = t^2, \quad g_3(t) = t^3, \quad (0, \infty)$ Independent

(2) Determine whether the indicated set of functions forms a fundamental solution set for the given ODE.

(a) $y_1 = x, \quad y_2 = \frac{1}{x} \quad x^2y''+xy'-y = 0, \quad x > 0$ yes

(b) $y_1 = xe^x, \quad y_2 = e^{2x} \quad y''-2y'+y = 0,$ no, y_2 doesn't solve it

(c) $y_1 = e^{2x}, \quad y_2 = e^{2x+1} \quad y''+y'-6y = 0,$ no, dependent

(d) $y_1 = e^{2x}, \quad y_2 = e^{-3x}, \quad y_3 = 1 \quad y'''+y''-6y' = 0,$ yes

(3) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0) = 0$. $i(t) = 50(1 - e^{-t/5})$

(4) An RC series circuit with resistance of 10 ohms and capacitance of 0.1 farads has electromotive force of $E(t) = 20te^{-t}$ applied to it. Find the charge on the capacitor $q(t)$ if $q(0) = 0$. $q(t) = t^2e^{-t}$

(5) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at

a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find the amount of salt $A(t)$ in the tank at time t. $\quad A(t) = 100 - 95e^{-t/100}$

(6) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes. $\quad A(30) = \frac{125}{32}$ lbs

(7) A population of bacteria experience exponential growth. If the initial population $P(0) = 1000$, and the population doubles every 4 hours, determine the population $P(t)$ for all $t > 0$. $P(t) = 1000e^{\left(\frac{\ln 2}{4}\right)t}$ for t in hours

(8) Solve each IVP.

(a) $\dfrac{dy}{dx} - \tan x\, y = \sin x, \quad y(0) = 1 \qquad y = \dfrac{1}{2}\sin^2 x \sec x + \sec x$

(b) $x\dfrac{dy}{dx} + 3\,y = \dfrac{1}{x^2(1+x^2)}, \quad y(1) = 0 \qquad y = \dfrac{\tan^{-1} x}{x^3} - \dfrac{\pi}{4x^3}$

(9) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y'' - xy' + y = 0 \quad x > 1, \quad y_1(x) = x \qquad y_2(x) = e^x$

(b) $x^2y'' + 3xy' - 3y = 0 \quad x > 0, \quad y_1(x) = x^{-3} \qquad y_2(x) = x$