Review for Exam 2
MATH 2306
Sections Covered: 4, 5, 6, 7, 8

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Solve each Bernoulli equation. Answers should be presented explicitly.

(a) \( y' + 3y = y^2 e^{3x} \)

(b) \( \frac{dy}{dx} + 4xy = 4x \sqrt{y} \)

(2) Determine whether the indicated set of functions forms a fundamental solution set for the given ODE.

(a) \( y_1 = xe^x \), \( y_2 = e^{2x} \) \( y'' - 2y' + y = 0 \),

(b) \( y_1 = e^{2x} \), \( y_2 = e^{2x+1} \) \( y'' + y' - 6y = 0 \),

(c) \( y_1 = e^{2x} \), \( y_2 = e^{-3x} \), \( y_3 = 1 \) \( y''' + y'' - 6y' = 0 \),

(3) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current \( i(t) \) if \( i(0) = 0 \).

(4) An RC series circuit with resistance of 10 ohms and capacitance of 0.1 farads has electromotive force of \( E(t) = 20te^{-t} \) applied to it. Find the charge on the capacitor \( q(t) \) if \( q(0) = 0 \).

(5) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find

\(^1\)Bernoulli
the amount of salt \( A(t) \) in the tank at time \( t \).

(6) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes.

(7) A population of bacteria experience exponential growth. If the initial population \( P(0) = 1000 \), and the population doubles every 4 hours, determine the population \( P(t) \) for all \( t > 0 \).

(8) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) \((x-1)y''-xy'+y = 0 \quad x > 1, \quad y_1(x) = e^x\)

(b) \(x^2y''+3xy'-3y = 0 \quad x > 0, \quad y_1(x) = x\)

(9) Find the general solution of the homogeneous equation.

(a) \(y''+6y'+9y = 0\)

(b) \(y''-36y = 0\)

(c) \(y^{(4)}+3y''-4y = 0\)

(10) Solve each IVP

(a) \(y''-3y'+2y = 0 \quad y(0) = 0, \quad y'(0) = 2\)

(b) \(y''+2y' = 0 \quad y(1) = 0, \quad y'(1) = 1\)
(c) \( y'' - 2y' + 5y = 0 \quad y(0) = 0, \quad y'(0) = 2 \)

(11) For each homogeneous equation, write out the characteristic equation. If the equation doesn’t have a characteristic equation, briefly state why.

(a) \( \frac{3}{dx^4} \frac{d^4 y}{dx^4} - 2 \frac{d^3 y}{dx^3} + \frac{dy}{dx} - 4y = 0 \)

(b) \( 4y'' + 2xy' + e^x y = 0 \)

(c) \( x^3 y''' + 2x^2 y'' - 4xy' + y = 0 \)

(d) \( y^{(6)} + 16y^{(4)} - 12y'' + y = 0 \)