

Review for Exam 2

MATH 2306

Sections Covered: 4¹, 5, 6, 7, 8

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Solve each Bernoulli equation. Answers should be presented explicitly.

(a) $y' + 3y = y^2 e^{3x}$

(b) $\frac{dy}{dx} + 4xy = 4x\sqrt{y}$

(2) Determine whether the indicated set of functions forms a fundamental solution set for the given ODE.

(a) $y_1 = xe^x, \quad y_2 = e^{2x} \quad y'' - 2y' + y = 0,$

(b) $y_1 = e^{2x}, \quad y_2 = e^{2x+1} \quad y'' + y' - 6y = 0,$

(c) $y_1 = e^{2x}, \quad y_2 = e^{-3x}, \quad y_3 = 1 \quad y''' + y'' - 6y' = 0,$

(3) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0) = 0$.

(4) An RC series circuit with resistance of 10 ohms and capacitance of 0.1 farads has electromotive force of $E(t) = 20te^{-t}$ applied to it. Find the charge on the capacitor $q(t)$ if $q(0) = 0$.

(5) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find

¹Bernoulli

the amount of salt $A(t)$ in the tank at time t .

(6) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes.

(7) A population of bacteria experience exponential growth. If the initial population $P(0) = 1000$, and the population doubles every 4 hours, determine the population $P(t)$ for all $t > 0$.

(8) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y'' - xy' + y = 0 \quad x > 1, \quad y_1(x) = e^x$

(b) $x^2y'' + 3xy' - 3y = 0 \quad x > 0, \quad y_1(x) = x$

(9) Find the general solution of the homogeneous equation.

(a) $y'' + 6y' + 9y = 0$

(b) $y'' - 36y = 0$

(c) $y^{(4)} + 3y'' - 4y = 0$

(10) Solve each IVP

(a) $y'' - 3y' + 2y = 0 \quad y(0) = 0, \quad y'(0) = 2$

(b) $y'' + 2y' = 0 \quad y(1) = 0, \quad y'(1) = 1$

(c) $y'' - 2y' + 5y = 0$ $y(0) = 0$, $y'(0) = 2$

(11) For each homogeneous equation, write out the characteristic equation. If the equation doesn't have a characteristic equation, briefly state why.

(a) $3\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + \frac{dy}{dx} - 4y = 0$

(b) $4y'' + 2xy' + e^x y = 0$

(c) $x^3 y''' + 2x^2 y'' - 4xy' + y = 0$

(d) $y^{(6)} + 16y^{(4)} - 12y'' + y = 0$