Solutions to Review for Exam 2

MATH 2306

Sections Covered: 4^1, 5, 6, 7, 8

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Solve each Bernoulli equation. Answers should be presented explicitly.

(a) $y'+3y = y^2 e^{3x}$ $y = \dfrac{e^{-3x}}{c-x}$

(c) $\dfrac{dy}{dx}+4xy = 4x\sqrt{y}$ $y = (1 + ce^{-x^2})^2$

(2) Determine whether the indicated set of functions forms a fundamental solution set for the given ODE.

(a) $y_1 = xe^x$, $y_2 = e^{2x}$ $y''-2y'+y = 0$,   no, y_2 doesn't solve it

(b) $y_1 = e^{2x}$, $y_2 = e^{2x+1}$ $y''+y'-6y = 0$,   no, dependent

(c) $y_1 = e^{2x}$, $y_2 = e^{-3x}$, $y_3 = 1$ $y'''+y''-6y' = 0$,   yes

(2) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0) = 0$. $i(t) = 50(1 - e^{-t/5})$

(3) An RC series circuit with resistance of 10 ohms and capacitance of 0.1 farads has electromotive force of $E(t) = 20te^{-t}$ applied to it. Find the charge on the capacitor $q(t)$ if $q(0) = 0$. $q(t) = t^2 e^{-t}$

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[1]Bernoulli

(4) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find the amount of salt $A(t)$ in the tank at time t. $A(t) = 100 - 95e^{-t/100}$

(5) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes. $A(30) = \frac{125}{32}$ lbs

(6) A population of bacteria experience exponential growth. If the initial population $P(0) = 1000$, and the population doubles every 4 hours, determine the population $P(t)$ for all $t > 0$.
   $P(t) = 1000e^{\left(\frac{\ln 2}{4}\right)t}$ for t in hours

(7) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y'' - xy' + y = 0$ $x > 1$, $y_1(x) = e^x$ $y_2(x) = x$

(b) $x^2 y'' + 3xy' - 3y = 0$ $x > 0$, $y_1(x) = x$ $y_2(x) = x^{-3}$

(8) Find the general solution of the homogeneous equation.

(a) $y'' + 6y' + 9y = 0$ $y = c_2 e^{-3x} + c_2 x e^{-3x}$

(b) $y'' - 36y = 0$ $y = c_1 e^{6x} + c_2 e^{-6x}$

(c) $y^{(4)} + 3y'' - 4y = 0$ $y = c_1 \cos(2x) + c_2 \sin(2x) + c_3 e^x + c_4 e^{-x}$

(9) Solve each IVP

(a) $y'' - 3y' + 2y = 0 \quad y(0) = 0, \quad y'(0) = 2$ $\textcolor{red}{y = 2e^{2x} - 2e^x}$

(b) $y'' + 2y' = 0 \quad y(1) = 0, \quad y'(1) = 1$ $\textcolor{red}{y = \dfrac{1}{2} - \dfrac{e^2}{2}e^{-2x}}$

(c) $y'' - 2y' + 5y = 0 \quad y(0) = 0, \quad y'(0) = 2$ $\textcolor{red}{y = e^x \sin(2x)}$

(10) For each homogeneous equation, write out the characteristic equation. If the equation doesn't have a characteristic equation, briefly state why.

(a) $3\dfrac{d^4y}{dx^4} - 2\dfrac{d^3y}{dx^3} + \dfrac{dy}{dx} - 4y = 0$ $\textcolor{red}{3m^4 - 2m^3 + m - 4 = 0}$

(b) $4y'' + 2xy' + e^x y = 0$ $\textcolor{red}{\text{none exists, it's not constant coefficient}}$

(c) $x^3 y''' + 2x^2 y'' - 4xy' + y = 0$ $\textcolor{red}{\text{none exists, it's not constant coefficient}}$

(d) $y^{(6)} + 16y^{(4)} - 12y'' + y = 0$ $\textcolor{red}{m^6 + 16m^4 - 12m^2 + 1 = 0}$