

Review for Exam I

MATH 2306 sec. 52

Sections Covered: 5, 6, 7, 8, 9

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0) = 0$.

(2) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find the amount of salt $A(t)$ in the tank at time t .

(3) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes.

(4) Determine whether the set of functions is linearly dependent or linearly independent on the indicated interval.

(a) $y_1(x) = e^{x+1}$, $y_2(x) = e^{x-1}$, $(-\infty, \infty)$

(b) $f_1(x) = \sin x$, $f_2(x) = \tan x$, $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(c) $g_1(t) = t$, $g_2(t) = t^2$, $g_3(t) = t^3$, $(0, \infty)$

(5) Find the general solution of the homogeneous equation.

(a) $y'' - 4y' + 5y = 0$

(b) $y'' + 6y' + 9y = 0$

(c) $y'' - 36y = 0$

(d) $y^{(4)} + 3y'' - 4y = 0$

(6) Solve the IVP

$$y'' - 3y' + 2y = 0 \quad y(0) = 0, \quad y'(0) = 2$$

(7) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y'' - xy' + y = 0 \quad x > 1, \quad y_1(x) = e^x$

(b) $x^2y'' + 3xy' - 3y = 0 \quad x > 0, \quad y_1(x) = x$

The remaining problems are from section 9.

(8) Find the general solution of the nonhomogeneous equation

$$y'' + 6y' + 9y = e^x + 3e^{-3x}$$

(9) Determine the form of the particular solution.

(a) $y'' - 4y' + 5y = x \cos 2x$

(b) $y'' + y = x^3 + e^x$

(c) $y'' - 4y' + 5y = xe^{2x} \sin x$

(d) $y'' - 2y' + y = 1 + e^x$