Review for Exam I

MATH 2306 sec. 52

Sections Covered: 5, 6, 7, 8, 9

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) An LR series circuit with inductance 20 henries and resistance 4 ohms has electromotive force of 200 volts applied to it. Find the current $i(t)$ if $i(0) = 0$. $i(t) = 50(1 - e^{-t/5})$

(2) A tank initially contains 500 L of salt water in which 5 kg of salt is dissolved. Suppose a brine solution containing 0.2 kg of salt per liter runs into the tank. The brine enters the tank at a rate of 5 L/min, and the well mixed solution is flowing out of the tank at the same rate. Find the amount of salt $A(t)$ in the tank at time t. $A(t) = 100 - 95e^{-t/100}$

(3) A large tank is partially filled with 100 gallons of fluid into which 10 pounds of salt is dissolved. Fresh water is pumped in at a rate of 6 gallons per minute, and the well mixed solution is pumped out at the slower rate of 4 gallons per minute. Determine the number of pounds of salt in the tank after 30 minutes. $A(30) = \frac{125}{32}$ lbs

(4) Determine whether the set of functions is linearly dependent or linearly independent on the indicated interval.

(a) $y_1(x) = e^{x+1}, \quad y_2(x) = e^{x-1}, \quad (-\infty, \infty)$ **Dependent**

(b) $f_1(x) = \sin x, \quad f_2(x) = \tan x, \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **Independent**

(c) $g_1(t) = t, \quad g_2(t) = t^2, \quad g_3(t) = t^3, \quad (0, \infty)$ **Independent**

(5) Find the general solution of the homogeneous equation.

(a) $y'' - 4y' + 5y = 0, \qquad y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x$

(b) $y''+6y'+9y=0 \qquad y=c_1e^{-3x}+c_2xe^{-3x}$

(c) $y''-36y=0 \qquad y=c_1e^{6x}+c_2e^{-6x}$

(d) $y^{(4)}+3y''-4y=0 \qquad y=c_1e^{x}+c_2e^{-x}+c_3\cos(2x)+c_4\sin(2x)$

(6) Solve the IVP

$y''-3y'+2y=0 \quad y(0)=0, \quad y'(0)=2 \qquad y=2e^{2x}-2e^{x}$

(7) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y''-xy'+y=0 \quad x>1, \quad y_1(x)=e^x, \qquad y_2(x)=x$

(b) $x^2y''+3xy'-3y=0 \quad x>0, \quad y_1(x)=x, \qquad y_2(x)=x^{-3}$

The remaining problems are from section 9.

(8) Find the general solution of the nonhomogeneous equation

$$y''+6y'+9y=e^x+3e^{-3x} \qquad y=c_1e^{-3x}+c_2xe^{-3x}+\frac{3}{2}x^2e^{-3x}+\frac{1}{16}e^x$$

(9) Determine the form of the particular solution.

(a) $y''-4y'+5y=x\cos 2x \qquad y_p=(Ax+B)\cos(2x)+(Cx+D)\sin(2x)$

(b) $y''+y=x^3+e^x \qquad y_p=Ax^3+Bx^2+Cx+D+Ee^x$

(c) $y''-4y'+5y=xe^{2x}\sin x \qquad y_p=(Ax^2+Bx)e^{2x}\cos x+(Cx^2+Dx)e^{2x}\sin x$

(d) $y''-2y'+y=1+e^x \qquad y_p=A+Bx^2e^x$