Review for Exam II

MATH 2306 (Ritter)

Sections Covered: 4.1, 4.2, 4.3, 4.4, (4.6 ?)

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Determine whether the set of functions is linearly dependent or linearly independent on the indicated interval.

(a) $y_1(x) = e^{x+1}, \quad y_2(x) = e^{x-1}, \quad (-\infty, \infty)$ **Dependent**

(b) $f_1(x) = \sin x, \quad f_2(x) = \tan x, \quad \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ **Independent**

(c) $g_1(t) = t, \quad g_2(t) = t^2, \quad g_3(t) = t^3, \quad (0, \infty)$ **Independent**

(2) Find the general solution of the homogeneous equation.

(a) $y'' - 4y' + 5y = 0, \qquad y = c_1 e^{2x} \cos x + c_2 e^{2x} \sin x$

(b) $y'' + 6y' + 9y = 0 \qquad y = c_1 e^{-3x} + c_2 x e^{-3x}$

(c) $y'' - 36y = 0 \qquad y = c_1 e^{6x} + c_2 e^{-6x}$

(d) $y^{(4)} + 3y'' - 4y = 0 \qquad y = c_1 e^{x} + c_2 e^{-x} + c_3 \cos(2x) + c_4 \sin(2x)$

(3) Solve the IVP

$y'' - 3y' + 2y = 0 \quad y(0) = 0, \quad y'(0) = 2 \qquad y = 2e^{2x} - 2e^{x}$

(4) Find the general solution of the nonhomogeneous equation

$$y'' + 6y' + 9y = e^x + 3e^{-3x} \qquad y = c_1e^{-3x} + c_2xe^{-3x} + \frac{3}{2}x^2e^{-3x} + \frac{1}{16}e^x$$

(5) Determine the form of the particular solution.

(a) $y''-4y'+5y = x\cos 2x \qquad y_p = (Ax+B)\cos(2x)+(Cx+D)\sin(2x)$

(b) $y''+y = x^3+e^x \qquad y_p = Ax^3+Bx^2+Cx+D+Ee^x$

(c) $y''-4y'+5y = xe^{2x}\sin x \qquad y_p = (Ax^2+Bx)e^{2x}\cos x+(Cx^2+Dx)e^{2x}\sin x$

(d) $y''-2y'+y = 1+e^x \qquad y_p = A+Bx^2e^x$

(6) Given one solution of the homogeneous equation, use reduction of order to find a second linearly independent solution.

(a) $(x-1)y''-xy'+y = 0 \quad x > 1, \quad y_1(x) = e^x, \qquad y_2(x) = x$

(b) $x^2y''+3xy'-3y = 0 \quad x > 0, \quad y_1(x) = x^{-3} \qquad y_2(x) = x$

The remaining questions involve section 4.6.

(7) Use 6(b) to solve the (non-homogeneous) initial value problem

$$x^2y'' + 3xy' - 3y = 15x^2, \quad y(1) = 0, \quad y'(1) = 0 \qquad y = \frac{3}{4}x^{-3} - \frac{15}{4}x + 3x^2$$

(8) Find the particular solution to the nonhomogeneous ODE using the method of variation of parameters.

$y''-4y'+4y = (x+1)e^{2x} \qquad y_p = \frac{1}{2}x^2e^{2x}+\frac{1}{6}x^3e^{2x}$

(9) Solve the IVP.

$y''-4y'+4y = (x+1)e^{2x} \quad y(0) = 3, \quad y'(0) = -3 \qquad y = 3e^{2x}-9xe^{2x}+\frac{1}{2}x^2e^{2x}+\frac{1}{6}x^3e^{2x}$