Practice for Exam 2 MATH 3260 sec. 57 & 58 Fall 2017

Sections Covered: 2.2, 2.3, 3.1, 3.2, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Evaluate the determinant of each matrix.

(a) $\begin{vmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 4 & 0 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -2 & 0 & -2 \end{vmatrix}$ (c) $\begin{vmatrix} 2 & 1 & -1 \\ 0 & k & 3 \\ 0 & 0 & k-1 \end{vmatrix}$

(2) Find all values of x such that $\det(A) = 0$ where

$$A = \begin{bmatrix} x-1 & 3 \\ 2 & x \end{bmatrix}$$

(3) Solve the following system of equations using a matrix inverse.

$$\begin{array}{rcrcl} 2x_1 & - & 4x_2 & = & 5 \\ -3x_1 & + & 2x_2 & = & 0 \end{array}$$

(4) Use the row reduction algorithm to find the inverse matrix if it exists.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Determine if the given set is a subspace of $\mathbb{R}^3$.

(a) $S = \{(x_1, x_2, x_3) | x_1 = x_2 + x_3\}$

(b) $S = \{(x_1, x_2, x_3) | x_1 - x_3 = 1\}$

(6) Let $\mathbb{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Determine if the following sets are subspaces of $\mathbb{P}_2$. (The symbol "$\in$" means "is an element of".)

(a) $S = \{at^2 + bt + c \,|\, a > 0\}$

(b) $S = \{\mathbf{p}(t) \in \mathbb{P}_2 \,|\, \mathbf{p}(1) = 0 \text{ and } \mathbf{p}(0) = 0\}$

(7) Find a matrix A such that the given set is Col$A$.

$$\left\{ \begin{bmatrix} b - c \\ 2b + c + d \\ 5c - 4d \\ d \end{bmatrix} : b, c, d \text{ in } \mathbb{R} \right\}$$

(8) Find a spanning set for Nul$A$ and a spanning set for Col$A$ where

$$A = \begin{bmatrix} 3 & 6 & 1 & -6 \\ 2 & 4 & 1 & -5 \end{bmatrix}$$

(9) The matrices A and B are row equivalent. Use this information to find bases for Nul$A$, Col$A$, and Row$A$. Determine both rank$A$ and dim(Nul$A$).

$$\text{(a)} \quad A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$(b)\quad A=\begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}, \quad B=\begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 & -13 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(10) Show that the given set is a basis for $\mathbb{R}^2$. Determine the change of coordinates matrix $P_{\mathcal{B}}$ and its inverse (use the order presented here). Then use this to find the indicated coordinate vectors.

$$\mathcal{B}=\left\{\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}\right\}$$

Determine $[\mathbf{x}]_{\mathcal{B}}$ for

$$(a)\quad \mathbf{x}=\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad (b)\quad \mathbf{x}=\begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad (c)\quad \mathbf{x}=\begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

(11) Determine the dimension of the indicated vector space.

(a) The subspace of $\mathbb{R}^4$ of vectors whose components sum to zero.

(b) The subspace of $\mathbb{P}_4$ consisting of polynomials of the form $\mathbf{p}(t) = at^4 + b(t^2 - t)$, for real numbers a and b.

(c) The null space of a 5×8 matrix with a rank of 4.

(d) The row space of an $m \times n$ matrix whose null space has dimension 6. (It may be necessary to express the answer in terms of m or n or both.)

Vector Space Axioms

A nonempty set $\mathbf{V}$ together, with operations called *addition* and *scalar multiplication*, is a vector space if the following properties hold for every $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbf{V}$ and scalars c and d in $\mathbb{R}$.

(1) $\mathbf{u} + \mathbf{v}$ is in $\mathbf{V}$

(2) $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$

(3) $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$

(4) There is an element denoted $\mathbf{0}$ in $\mathbf{V}$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for every $\mathbf{u}$ in $\mathbf{V}$.

(5) For each element $\mathbf{u}$ in $\mathbf{V}$ there is an element denoted $-\mathbf{u}$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(6) $c\mathbf{u}$ is in $\mathbf{V}$ for each scalar c

(7) $(cd)\mathbf{u} = c(d\mathbf{u})$

(8) $(c + d)\mathbf{u} = c\mathbf{u} + d\mathbf{u}$

(9) $c(\mathbf{u} + \mathbf{v}) = c\mathbf{u} + c\mathbf{v}$

(10) $1\mathbf{u} = \mathbf{u}$