Solutions to Practice for Exam 2 MATH 3260 sec. 57 & 58 Fall 2017

Sections Covered: 2.2, 2.3, 3.1, 3.2, 4.1, 4.2, 4.3, 4.4, 4.5, 4.6

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Evaluate the determinant of each matrix.

(a) $\begin{vmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 4 & 0 & 2 \end{vmatrix} = 0$ (b) $\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -2 & 0 & -2 \end{vmatrix} = -6$ (c) $\begin{vmatrix} 2 & 1 & -1 \\ 0 & k & 3 \\ 0 & 0 & k-1 \end{vmatrix} = 2k(k-1)$

(2) Find all values of x such that $\det(A) = 0$ where

$$A = \begin{bmatrix} x-1 & 3 \\ 2 & x \end{bmatrix} \quad x = -2, \quad \text{or} \quad x = 3$$

(3) Solve the following system of equations using a matrix inverse.

$$\begin{aligned} 2x_1 &- 4x_2 &= 5 \\ -3x_1 &+ 2x_2 &= 0 \end{aligned} \quad \left(-\frac{5}{4}, -\frac{15}{8}\right)$$

(4) Use the row reduction algorithm to find the inverse matrix if it exists.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Determine if the given set is a subspace of $\mathbb{R}^3$. **The answers I'm giving here would <u>NOT</u> be sufficient if you gave them on a test. You are expected to justify your claims. I'm just providing you a way to double check your understanding.**

(a) $S = \{(x_1, x_2, x_3) | x_1 = x_2 + x_3\}$ It is. Check the three criteria, or identify a spanning set.

(b) $S = \{(x_1, x_2, x_3) | x_1 - x_3 = 1\}$ It is not. In particular, $\mathbf{0}$ is not in it.

(6) Let $\mathbb{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Determine if the following sets are subspaces of $\mathbb{P}_2$. (The symbol "$\in$" means "is an element of".) **Again, I'd expect justification for such a claim on an exam.**

(a) $S = \{at^2 + bt + c \,|\, a > 0\}$ It is not. In particular, $\mathbf{0}$ is not in it.

(b) $S = \{\mathbf{p}(t) \in \mathbb{P}_2 \,|\, \mathbf{p}(1) = 0 \text{ and } \mathbf{p}(0) = 0\}$ It is. Check the three criteria, or identify a spanning set.

(7) Find a matrix A such that the given set is Col$A$.

$$\left\{ \begin{bmatrix} b - c \\ 2b + c + d \\ 5c - 4d \\ d \end{bmatrix} : b, c, d \text{ in } \mathbb{R} \right\} \quad A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

(8) Find a spanning set for Nul$A$ and a spanning set for Col$A$ where

$$A = \begin{bmatrix} 3 & 6 & 1 & -6 \\ 2 & 4 & 1 & -5 \end{bmatrix} \quad \text{Nul}A = \text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}, \quad \text{Col}A = \text{Span}\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$$

(9) The matrices A and B are row equivalent. Use this information to find bases for Nul$A$, Col$A$, and Row$A$. Determine both rank$A$ and dim(Nul$A$).

(a) $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\text{Nul}A = \text{Span}\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}, \quad \text{Col}A = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -3 \\ 0 \end{bmatrix} \right\}$$

$$\text{Row}A = \text{Span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 4 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -5 \end{bmatrix} \right\}, \quad \text{rank}A = 3, \quad \dim(\text{Nul}A) = 2$$

(b) $A = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 0 & 1 & -1 & 0 & -3 & -1 \\ 0 & 0 & 1 & 1 & -13 & -1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$$\text{Nul}A = \text{Span}\left\{ \begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}, \quad \text{Col}A = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 6 \\ 0 \\ -1 \end{bmatrix} \right\}$$

$$\text{Row}A = \text{Span}\left\{\begin{bmatrix}1\\1\\-2\\0\\1\\-2\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\0\\-3\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\1\\1\\-13\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}\right\}, \quad \text{rank}A = 5, \quad \dim(\text{Nul}A) = 1$$

(10) Show that the given set is a basis for $\mathbb{R}^2$. Determine the change of coordinates matrix $P_{\mathcal{B}}$ and its inverse (use the order presented here). Then use this to find the indicated coordinate vectors.

$$\mathcal{B} = \left\{\begin{bmatrix}5\\1\end{bmatrix}, \begin{bmatrix}2\\2\end{bmatrix}\right\}, \quad P_{\mathcal{B}} = \begin{bmatrix}5 & 2\\1 & 2\end{bmatrix}, \quad P_{\mathcal{B}}^{-1} = \frac{1}{8}\begin{bmatrix}2 & -2\\-1 & 5\end{bmatrix}$$

Determine $[\mathbf{x}]_{\mathcal{B}}$ for

(a) $\mathbf{x} = \begin{bmatrix}5\\1\end{bmatrix}$, $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix}1\\0\end{bmatrix}$ (b) $\mathbf{x} = \begin{bmatrix}2\\2\end{bmatrix}$, $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix}0\\1\end{bmatrix}$ (c) $\mathbf{x} = \begin{bmatrix}-2\\3\end{bmatrix}$ $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix}-\frac{5}{4}\\\frac{17}{8}\end{bmatrix}$

(11) Determine the dimension of the indicated vector space.

(a) The subspace of $\mathbb{R}^4$ of vectors whose components sum to zero. 3

(b) The subspace of $\mathbb{P}_4$ consisting of polynomials of the form $\mathbf{p}(t) = at^4 + b(t^2 - t)$, for real numbers a and b. 2

(c) The null space of a 5×8 matrix with a rank of 4. 4

(d) The row space of an $m \times n$ matrix whose null space has dimension 6. (It may be necessary to express the answer in terms of m or n or both.) $n - 6$