Practice for Exam 2 (Ritter) MATH 3260 Spring 2018

Sections Covered: 1.8, 1.9, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 4.1, 4.2, 4.3

These practice problems are intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Determine whether the transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ is a linear transformation. Justify your conclusion.

$$T(x_1, x_2, x_3) = (x_1 + 1, 0, 0, x_2 - x_3)$$

(2) Show that T is a linear transformation by finding a matrix that implements the mapping. (The terms x_1, x_2 etc. are entries in a vector as opposed to vectors themselves.)

$$T(x_1, x_2) = (x_1 - 2x_2, 3x_2, 0, 4x_1 + x_2)$$

(3) Show that if T is a linear transformation, then it is necessarily true that $T(\mathbf{0}) = \mathbf{0}$.

(4) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear tranformation defined by

$$T(x_1, x_2, x_3) = (x_1 - 3x_3, 2x_1 + 2x_2).$$

(a) Identify the domain and codomain.

(b) Determine if T is one to one.

(c) Determine if T is onto.

(d) Is $(1, 1)$ in the domain of T? If so, find its image. Is $(1, 1)$ in the range of T? Why or why not?

(5) Consider the matrices given. Compute each expression if it exists. If it doesn't exist, state why.

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 & -3 \\ 1 & -1 & 0 \\ 3 & 4 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 4 & -8 \end{bmatrix}$$

(a) $A + 2B$

(b) A^T

(c) $B + C$

(d) AC

(e) AC^T

(f) C^TC

(6) Evaluate the determinant of each matrix.

(a) $\begin{vmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 4 & 0 & 2 \end{vmatrix}$ (b) $\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -2 & 0 & -2 \end{vmatrix}$ (c) $\begin{vmatrix} 2 & 1 & -1 \\ 0 & k & 3 \\ 0 & 0 & k-1 \end{vmatrix}$

(7) Find all values of x such that $\det(A) = 0$ where $A = \begin{bmatrix} x-1 & 3 \\ 2 & x \end{bmatrix}$.

(8) Solve the following system of equations twice, first using a matrix inverse, and then using Crammer's rule.

$$\begin{aligned} 2x_1 &- 4x_2 &= 5 \\ -3x_1 &+ 2x_2 &= 0 \end{aligned}$$

(9) Use the row reduction algorithm to find the inverse matrix if it exists.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

(10) Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (1, 0, -1)$, $\mathbf{v} = (2, 1, 1)$, and $\mathbf{w} = (0, -3, 3)$.

(11) Determine if the given set is a subspace of $\mathbb{R}^3$.

(a) $S = \{(x_1, x_2, x_3) | x_1 = x_2 + x_3\}$

(b) $S = \{(x_1, x_2, x_3) | x_1 - x_3 = 1\}$

(12) Let $\mathbb{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Determine if the following sets are subspaces of $\mathbb{P}_2$.

(a) $S = \{at^2 + bt + c \,|\, a > 0\}$

(b) $S = \{\mathbf{p}(t) \text{ in } \mathbb{P}_2 \,|\, \mathbf{p}(1) = 0 \text{ and } \mathbf{p}(0) = 0\}$

(13) Find a matrix A such that the given set is Col$A$.

$$\left\{ \begin{bmatrix} b - c \\ 2b + c + d \\ 5c - 4d \\ d \end{bmatrix} : b, c, d \text{ in } \mathbb{R} \right\}$$

(14) Find a spanning set for Nul$A$ and a spanning set for Col$A$ where

(a) $A = \begin{bmatrix} 3 & 6 & 1 & -6 \\ 2 & 4 & 1 & -5 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$