Practice for Exam 2 (Ritter) MATH 3260 Spring 2018

Sections Covered: 1.8, 1.9, 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 4.1, 4.2, 4.3

These practice problems are intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Determine whether the transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ is a linear transformation. Justify your conclusion.

$$T(x_1, x_2, x_3) = (x_1 + 1, 0, 0, x_2 - x_3)$$ No. For one thing, $T(\mathbf{0}) \neq \mathbf{0}$.

(2) Show that T is a linear transformation by finding a matrix that implements the mapping. (The terms x_1, x_2 etc. are entries in a vector as opposed to vectors themselves.)

$$T(x_1, x_2) = (x_1 - 2x_2, 3x_2, 0, 4x_1 + x_2)$$ The standard matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \\ 0 & 0 \\ 4 & 1 \end{bmatrix}$

(3) Show that if T is a linear transformation, then it is necessarily true that $T(\mathbf{0}) = \mathbf{0}$. One argument can be made by taking any $\mathbf{x}$ in the domain. By definition of scalar multiplication, $\mathbf{0} = 0\mathbf{x}$ (the zero vector in the domain of T). Then using the linearity properties, $T(\mathbf{0}) = T(0\mathbf{x}) = 0T(\mathbf{x}) = \mathbf{0}$ where the final term is the zero vector in the codomain.

(4) Let $T : \mathbb{R}^n \longrightarrow \mathbb{R}^m$ be a linear tranformation defined by

$$T(x_1, x_2, x_3) = (x_1 - 3x_3, 2x_1 + 2x_2).$$

(a) Identify the domain and codomain. $\mathbb{R}^3$ and $\mathbb{R}^2$, respectively

(b) Determine if T is one to one. No. Note that $T(3, -3, 1) = (0, 0)$.

(c) Determine if T is onto. Yes. Show for example that $\{T(\mathbf{e}_2), T(\mathbf{e}_3)\}$ spans $\mathbb{R}^2$.

(d) Is $(1, 1)$ in the domain of T? If so, find its image. Is $(1, 1)$ in the range of T? Why or why not? No to the first since it's not in $\mathbb{R}^3$. Yes to the second by virtue of the answer to part (c) above.

(5) Consider the matrices given. Compute each expression if it exists. If it doesn't exist, state why.

$$A = \begin{bmatrix} 2 & -2 & 1 \\ 1 & 1 & -1 \\ 0 & 1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 & -3 \\ 1 & -1 & 0 \\ 3 & 4 & -3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 4 & -8 \end{bmatrix}$$

(a) $A + 2B$ $\quad \begin{bmatrix} 6 & 8 & -5 \\ 3 & -1 & -1 \\ 6 & 9 & -3 \end{bmatrix}$

(b) A^T $\quad \begin{bmatrix} 2 & 1 & 0 \\ -2 & 1 & 1 \\ 1 & -1 & 3 \end{bmatrix}$

(c) $B + C$ This is undefined since the matrices are not of the same size.

(d) AC This is undefined. C does not have the same number of rows as A does columns.

(e) AC^T $\quad \begin{bmatrix} -6 & -12 \\ 7 & 14 \\ -10 & -20 \end{bmatrix}$

(f) C^TC $\quad \begin{bmatrix} 5 & 10 & -20 \\ 10 & 20 & -40 \\ -20 & -40 & 80 \end{bmatrix}$ As an aside, note that $(C^TC)^T = C^TC$ as expected.

There is a necessary symmetry to the entries of this product.

(6) Evaluate the determinant of each matrix.

(a) $\begin{vmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 4 & 0 & 2 \end{vmatrix} = 0$ (b) $\begin{vmatrix} 2 & 1 & -1 \\ 3 & 2 & 0 \\ -2 & 0 & -2 \end{vmatrix} = -6$ (c) $\begin{vmatrix} 2 & 1 & -1 \\ 0 & k & 3 \\ 0 & 0 & k-1 \end{vmatrix} = 2k(k-1)$

(7) Find all values of x such that $\det(A) = 0$ where $A = \begin{bmatrix} x-1 & 3 \\ 2 & x \end{bmatrix}$. $x = -2$, or $x = 3$

(8) Solve the following system of equations twice, first using a matrix inverse, and then using Crammer's rule.

$$\begin{aligned} 2x_1 &- 4x_2 = 5 \\ -3x_1 &+ 2x_2 = 0 \end{aligned} \qquad \left(-\frac{5}{4}, -\frac{15}{8}\right)$$

(9) Use the row reduction algorithm to find the inverse matrix if it exists.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(10) Find the volume of the parallelepiped determined by the vectors $\mathbf{u} = (1, 0, -1)$, $\mathbf{v} = (2, 1, 1)$, and $\mathbf{w} = (0, -3, 3)$. $V = \det([\mathbf{u}\ \mathbf{v}\ \mathbf{w}]) = 12$

(11) Determine if the given set is a subspace of $\mathbb{R}^3$. **The answers I'm giving here would <u>NOT</u> be sufficient if you gave them on a test. You are expected to justify your claims. I'm just providing you a way to double check your understanding.**

(a) $S = \{(x_1, x_2, x_3) | x_1 = x_2 + x_3\}$ It is. Check the three criteria, or identify a spanning set.

(b) $S = \{(x_1, x_2, x_3) | x_1 - x_3 = 1\}$ It is not. In particular, **0** is not in it.

(12) Let $\mathbb{P}_2$ denote the set of all polynomials of degree less than or equal to 2. Determine if the following sets are subspaces of $\mathbb{P}_2$. **Again, I'd expect justification for such a claim on an exam.**

(a) $S = \{at^2 + bt + c \,|\, a > 0\}$ It is not. In particular, **0** is not in it.

(b) $S = \{\mathbf{p}(t) \in \mathbb{P}_2 \,|\, \mathbf{p}(1) = 0 \text{ and } \mathbf{p}(0) = 0\}$ It is. Check the three criteria, or identify a spanning set.

(13) Find a matrix A such that the given set is Col$A$.

$$\left\{ \begin{bmatrix} b-c \\ 2b+c+d \\ 5c-4d \\ d \end{bmatrix} : b, c, d \text{ in } \mathbb{R} \right\} \quad A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 1 \\ 0 & 5 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

(14) Find a spanning set for Nul$A$ and a spanning set for Col$A$ where

(a) $A = \begin{bmatrix} 3 & 6 & 1 & -6 \\ 2 & 4 & 1 & -5 \end{bmatrix}$ $\quad \text{Nul}A = \text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 1 \end{bmatrix} \right\}, \quad \text{Col}A = \text{Span}\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

(b) $A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}$ $\quad \text{Nul}A = \text{Span}\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\},$

$$\text{Col}A = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -3 \\ 0 \end{bmatrix} \right\}$$

Note that in the above, if the term Span was not put in front of the curly brackets, the answer would be **WRONG**. Be careful to use proper notation. Expecting me to read your mind and just *know what you were thinking* is risky (and perhaps delusional).