Solutions to Review for Exam III

MATH 1112 sections 54 Spring 2019

Sections Covered: 6.4, 6.1, 6.2, 6.3, 6.5, 6.6, 7.4 (In *Miller*: 5.1–5.7 all)

Calculator Policy: Calculator use may be allowed on part of the exam. When instructions call for an **exact** solution, that indicates that a decimal approximation will not be accepted.

Problems marked with a blue asterisk * have detailed.

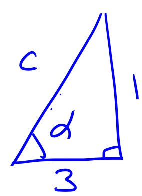
(1) Given one trigonometric value of an acute angle, find the remaining five trigonometric values.

$\cot\alpha = \frac{adj}{opp}$ Set $adj = 3$, $opp = 1$

(a) $\cot\alpha = 3$ *

$c^2 = 1^2 + 3^2 = 10$ $c = \sqrt{10}$

(b) $\sec\beta = \dfrac{7}{2}$



$\sin\alpha = \frac{1}{\sqrt{10}}$ $\csc\alpha = \sqrt{10}$ $\tan\alpha = \frac{1}{3}$

(c) $\sin\sigma = \dfrac{12}{13}$

$\cos\alpha = \frac{3}{\sqrt{10}}$ $\sec\alpha = \frac{\sqrt{10}}{3}$

See the end for (a) worked out.

$$\text{(b)}\quad \sin\beta = \frac{\sqrt{45}}{7},\quad \cos\beta = \frac{2}{7},\quad \tan\beta = \frac{\sqrt{45}}{2},\quad \cot\beta = \frac{2}{\sqrt{45}},\quad \csc\beta = \frac{7}{\sqrt{45}}$$

$$\text{(c)}\quad \sec\sigma = \frac{13}{5},\quad \cos\sigma = \frac{5}{13},\quad \tan\sigma = \frac{12}{5},\quad \cot\sigma = \frac{5}{12},\quad \csc\sigma = \frac{13}{12}$$

(2) Evaluate each expression exactly without a calculator.

(a) $\sin 30^\circ \cos 45^\circ = \dfrac{1}{2\sqrt{2}}$

(b) $\csc 60^\circ = \dfrac{2}{\sqrt{3}}$

(c) $\sin 60^\circ - 2\sin 30^\circ \cos 30^\circ = 0$

(3) Suppose the angle θ has terminal side in quadrant III when in standard position and that $\tan\theta = \frac{7}{6}$ determine the remaining five trigonometric values of θ. $\sin\theta = -\frac{7}{\sqrt{85}}$, $\cos\theta = -\frac{6}{\sqrt{85}}$, the others are readily deduced from here.

(4) A regular pentagon is inscribed in a circle of radius 10. Find the perimeter of the pentagon.

* See pages @ the end

(5) From a hot air balloon 2 km high, the angles of depression of two towns in line with the balloon and on the same side of the balloon are $81°$ and $13°$. How far apart are the towns (to the nearest km)? *

(6) Evaluate each trigonometric expression exactly if it exists. (Check with a calculator, but be able to do this without one. You can be sure I will ask you to do so on an exam.)

(a) $\cos\left(\frac{3\pi}{2}\right) = 0$ (b) $\cot(2\pi)$ undefined (c) $\csc\left(\frac{5\pi}{6}\right) = 2$

(d) $\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$ (e) $\tan\left(\frac{3\pi}{4}\right) = -1$ (f) $\cos\left(\frac{5\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

(g) $\sec\left(\frac{5\pi}{2}\right)$ undefined (h) $\sec\left(\frac{2\pi}{3}\right) = -2$ (i) $\tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$

(7) State the domain and range of each of the six trigonometric functions. Use interval notation or set builder notation.

Function	Domain	Range
$y = \sin x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \cos x$	$(-\infty, \infty)$	$[-1, 1]$
$y = \tan x$	$\left\{x : x \neq \frac{\pi}{2} + k\pi, \text{ for integer } k\right\}$	$(-\infty, \infty)$
$y = \csc x$	$\{x : x \neq k\pi \text{ for integer } k\}$	$(-\infty, -1] \cup [1, \infty)$
$y = \sec x$	$\left\{x : x \neq \frac{\pi}{2} + k\pi, \text{ for integer } k\right\}$	$(-\infty, -1] \cup [1, \infty)$
$y = \cot x$	$\{x : x \neq k\pi \text{ for integer } k\}$	$(-\infty, \infty)$

(8) Identify the amplitude and period of each of each function.

(a) $f(x) = -3\cos\left(\frac{x}{2}\right) - 2$ (b) $g(x) = 4 - 4\sin\left(\pi x + \frac{\pi}{6}\right)$ (c) $F(x) = 4\sin\left(\frac{\pi}{4} - 2x\right)$

For $y = a\cos(bx - c) + d$ (or the version with a sine function), amplitude is $|a|$, and period is $\frac{2\pi}{|b|}$. So (a) has amplitude 3 and period 4π; (b) has amplitude 4 and period 2; (c) has amplitude 4 and period π.

(9) State the domain and the range of each of $f(x) = \sin^{-1}(x)$, $g(x) = \cos^{-1}(x)$ and $H(x) = \tan^{-1}(x)$ using interval notation.

Function	Domain	Range
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(10) Evaluate each expression exactly if it exists. If it doesn't exist, state why.

(a) $\sin\left(\sin^{-1} 0.02\right) = 0.02$

(b) $\sin^{-1}(\sin 0.02) = 0.02$

(c) $\sin^{-1}[\sin(\pi)] = 0$ $\sin\pi = 0$ and $\sin^{-1} 0 = 0$

(d) $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right] = \frac{\pi}{4}$, $\cos\left(-\frac{pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\cos^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}$

(e) $\cos(\tan^{-1} 4) = \frac{1}{\sqrt{17}}$ *

(f) $\csc\left[\cos^{-1}\left(\frac{2}{3}\right)\right] = \frac{3}{\sqrt{5}}$ Draw a triangle in quadrant I with adjacent leg 2 and hypotenuse 3.

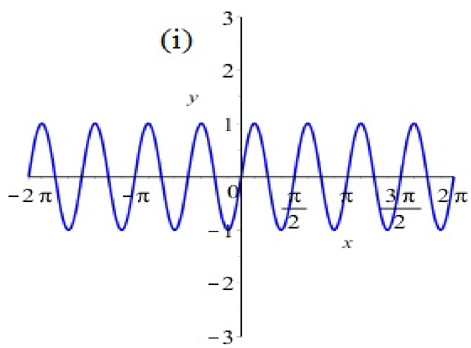
(11) Plot at least two full periods of each of $y = \sin x$, $y = \cos x$, and $y = \tan x$. Plot these and compare your graphs to the lecture slides or to an image you create in Desmos, Wolfram Alpha, or a graphing calculator.

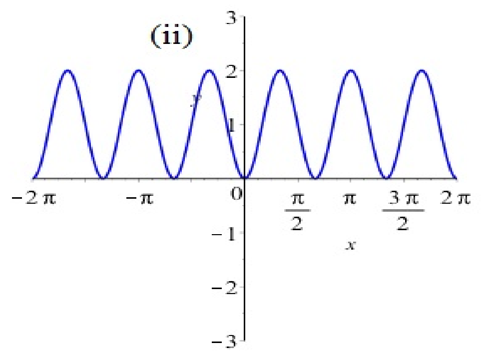
(12) Match the following functions with the plots shown. Note that not all of the functions will be used.

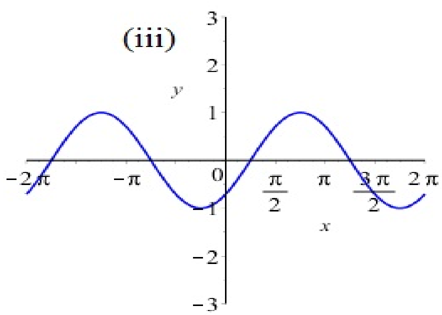
(a) $f(x) = 2 - \cos\left(x + \frac{\pi}{4}\right)$ (vi) (b) $f(x) = \sin(4x)$ (i) (c) $f(x) = -2\sin(2x) + 1$ NA

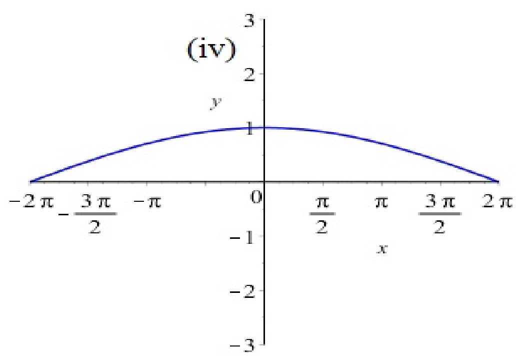
(d) $f(x) = -3\cos x + 1$ NA (e) $f(x) = -\cos(3x) + 1$ (ii) (f) $f(x) = \frac{1}{2}\sin(2x) - 2$ (v)

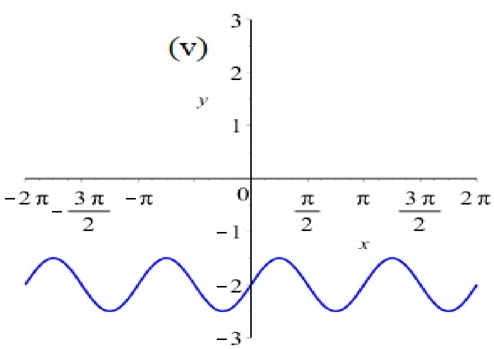
(g) $f(x) = 2 + \cos\left(\frac{\pi x}{4} - \frac{\pi}{2}\right)$ NA (h) $f(x) = \cos\left(\frac{x}{4}\right)$ (iv) (i) $f(x) = \sin\left(x - \frac{\pi}{4}\right)$ (iii)

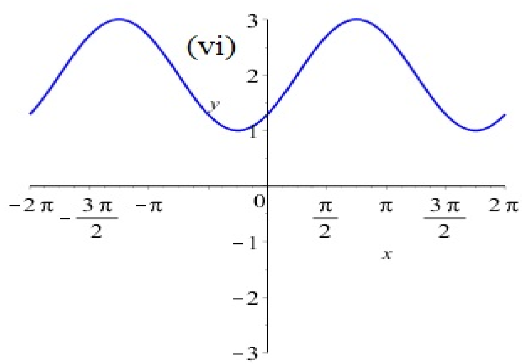

(i)
y
x
3
2
1
−1
−2
−3
−2 π
−π
0
π/2
π
3π/2
2 π


(ii)
y
x
3
2
1
−1
−2
−3
−2 π
−π
0
π/2
π
3π/2
2 π

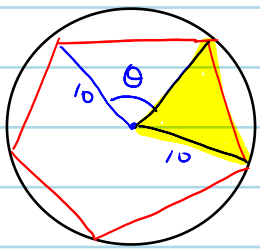

(iii)
y
x
3
2
1
−1
−2
−3
−2 π
−π
0
π/2
π
3π/2
2 π


(iv)
y
x
3
2
1
−1
−2
−3
−2 π
−3π/2
−π
0
π/2
π
3π/2
2 π


(v)
y
x
3
2
1
−1
−2
−3
−2 π
−3π/2
−π
0
π/2
π
3π/2
2 π

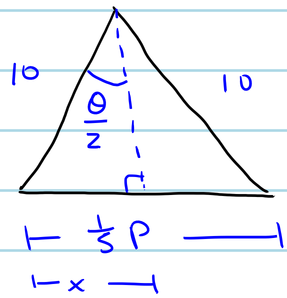

(vi)
y
x
3
2
1
−1
−2
−3
−2 π
−3π/2
−π
0
π/2
π
3π/2
2 π

4) Angle $\theta = \frac{360^\circ}{5} = 72^\circ$



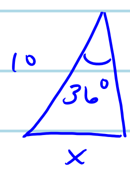
(5 equal angles in the circle)

Consider the yellow triangle



The base is $\frac{1}{5}$ th the perimeter, one of the 5 sides

From the 72° angle, drop a perpendicular to get a right triangle. $\frac{1}{2}(72^\circ) = 36^\circ$

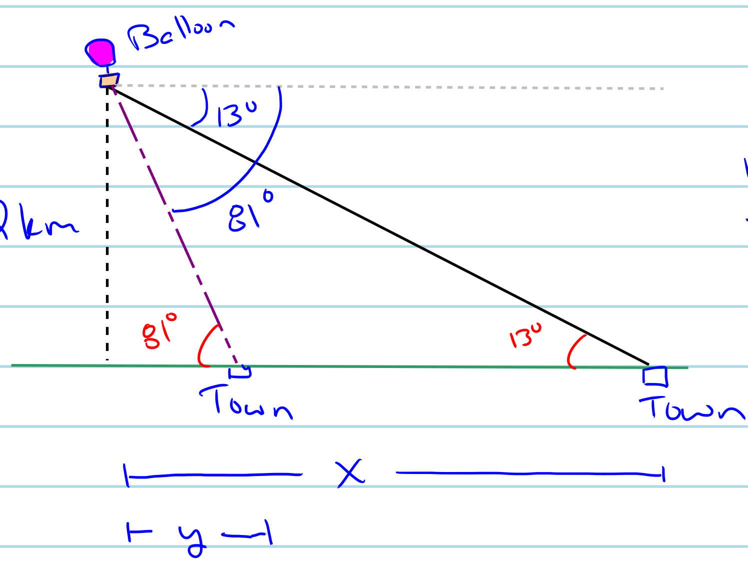


$\frac{x}{10} = \sin 36^\circ \Rightarrow x = 10\sin 36^\circ$

Now $x = \frac{1}{2}\left(\frac{1}{5}P\right) = \frac{1}{10}P$ so to get P, multiply by 10

$P = 10x = 10\left(10\sin 36^\circ\right) = 100\sin 36^\circ \approx 58.8$ units

5)



Here's a diagram

The distance between towns is $x - y$ from this picture

From the diagram

$$\frac{x}{2\text{ km}} = \cot 13^\circ \quad , \quad \frac{y}{2\text{ km}} = \cot 81^\circ$$

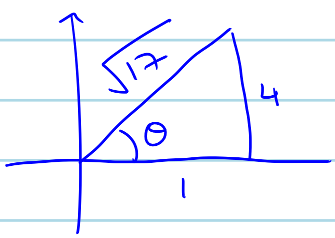
$x = 2\cot 13^\circ$ km $\quad y = 2\cot 81^\circ$ km

The distance between towns is

$$d = x - y = 2(\cot 13^\circ - \cot 81^\circ)\text{ km} \approx 8.3\text{ km}$$

10 e) $\cos(\tan^{-1} 4)$ Let $\theta = \tan^{-1} 4$. Then $\tan\theta = 4$ and $-\pi/2 < \theta < \pi/2$

Since $\tan\theta = 4 > 0$, θ is in quad I



Draw a representative triangle opp $= 4$, adj $= 1$

$$\text{hyp} = \sqrt{4^2 + 1^2} = \sqrt{17}$$

Then $\cos\theta = \frac{1}{\sqrt{17}}$, that is $\cos(\tan^{-1} 4) = \frac{1}{\sqrt{17}}$