**Review for Exam III**

**MATH 1113 sections 51 & 52 Fall 2018**

Sections Covered: 2.2 (diff. quotient), 2.1 (piecewise fnct), 5.1, 5.2, 5.3, 5.4, 5.5, 6.1 & 6.2

**Calculator Policy:** Calculator use may be allowed on part of the exam. When instructions call for an **exact** solution, that indicates that a decimal approximation will not be accepted.

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) Let $f(x) = \begin{cases} 2x-1, & -2 \le x \le 1 \\ 0, & 1 < x < 2 \\ 1, & x = 2 \\ 5-x^2, & 2 < x \le 3 \end{cases}$ Evaluate each of the following if possible. If a quantity doesn't exist, you can write "DNE." Where applicable, assume that $0 < h < 0.1$.

(a) $f(0)$

(b) $f\left(\frac{5}{2}\right)$

(c) $f(4)$

(d) $f(1+h)$

(e) $f(1-h)$

(f) $f(2+h)$

(2) Provide a sketch of each piecewise defined function. Identify the domain and range of each function.

(a) $f(x) = \begin{cases} 2x-1, & -2 \le x \le 1 \\ 0, & 1 < x < 2 \\ 1, & x = 2 \\ 5-x^2, & 2 < x \le 3 \end{cases}$

(b) $g(x) = \begin{cases} x+2, & -3 < x < -1 \\ x^2, & -1 < x < 1 \\ 3-x, & 1 \le x \le 3 \end{cases}$

(c) $h(x) = \begin{cases} e^{-x}, & -1 \le x \le 0 \\ \ln(x+1), & 0 < x \end{cases}$

(3) For each function and given value for $a$, evaluate the difference quotient $\dfrac{f(a+h)-f(a)}{h}$. Simplify your answer.

(a) $f(x) = 2x^2 - x, \quad \text{for} \quad a = -1$

(b) $f(x) = \dfrac{1}{x^2+3}, \quad \text{for} \quad a = 0$

(4) For each function given in exercise (3), evaluate $\dfrac{f(x+h)-f(x)}{h}$ for any $x$ in the domain of the function. Simplify to the extent possible.

(5) Let $y = \log_a(M)$ so that $a^y = M$. Take the logarithm base $b$ of both sides of the exponential equation, and using logarithm properties derive the change of base formula. (That is, show that $\log_b(M) = \frac{\log_a(M)}{\log_a(b)}$.)

(6) Identify each statement as true or false. (Full disclosure, some of these statements are embarrassingly ludicrous.)

(a) $\dfrac{\ln(x)}{x} = \ln$

(b) $\log_4(x) = \dfrac{\log_5(x)}{\log_5(4)}$

(c) $(e^x)^2 = e^{2x}$

(d) $\ln x = \dfrac{1}{x}$

(e) $\log_a(x-y) = \dfrac{\log_a(x)}{\log_a(y)}$

(f) $\log(8^9) = 9\log(8)$

(g) $e^{9x} = 9e^x$

(7) Each of the following functions is one to one on the indicated interval. Identify the inverse function.

(a) $f(x) = \dfrac{5x+3}{x-4}$

(b) $g(x) = 3x^5 + 7$

(c) $S(x) = e^{2x^3}$

(8) Use composition to show that the given functions are inverses.

$$f(x) = \sqrt[5]{\frac{x-1}{2x}} \quad \text{and} \quad f^{-1}(x) = \frac{1}{1-2x^5}$$

(9) Evaluate each expression without a calculator

(a) $\log_3(1)$ (b) $\log_2 \frac{1}{32}$ (c) $\ln \sqrt{e}$

(d) $\log(0.0001)$ (e) $\log_4(2^7)$ (f) $\log_\pi \pi$

(10) Express as a single logarithm. Simplify if possible.

(a) $4 \ln x + \frac{1}{3} \ln y - 2 \ln z$ (b) $\log_2(x^3 - 8) - \log_2(x^2 + 2x + 4)$

(11) Expand as a sum or difference of logarithms.

(a) $\ln \sqrt[4]{wr^2}$ (b) $\log \sqrt[3]{\frac{M^2}{N}}$

(12) Produce a plot of each function. Label any asymptotes and intercepts.

(a) $y = e^{x-1}$ (b) $f(t) = \ln(-t)$

(c) $g(x) = e^x + 2$ (d) $y = \log_{1/2} x$

(13) Solve each equation. Obtain an exact solution.

(a) $\log_3(x) + \log_3(x+1) = \log_3(2) + \log_3(x+3)$

(b) $\log_3(x^2 + x) = \log_3(2) + \log_3(x+3)$

(c) $e^x + e^{-x} = 3$

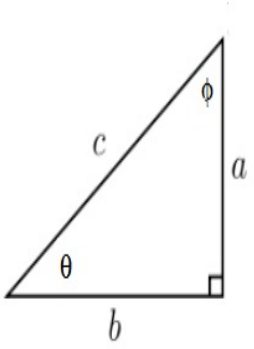
(d) $5^{x+1} = 3^{2x-1}$

(14) Given one trigonometric value of an acute angle, find the remaining five trigonometric values.

(a) $\cot \alpha = 3$

(b) $\sec \beta = \frac{7}{2}$

(c) $\sin \sigma = \frac{12}{13}$




(15) The variables used in this problem are defined in the figure above Use the given information to solve for the remaining side lengths and indicated trigonometric values.

(i) $c = 6$ and $\sin\theta = \dfrac{2}{3}$. Find $a$, $b$, $\cos\theta$ and $\tan\theta$.

(ii) $a = 1$ and $\tan\phi = 5$. Find $b$, $c$, $\sin\theta$ and $\sin\phi$.

(iii) $b = 4$ and $\cos\phi = \dfrac{1}{\sqrt{5}}$. Find $a$, $c$, $\sin\phi$ and $\tan\phi$.

(16) Evaluate each expression exactly without a calculator.

(a) $\sin 30^\circ \cos 45^\circ$

(b) $\csc 60^\circ$

(c) $\sin 60^\circ - 2\sin 30^\circ \cos 30^\circ$

(17) A regular pentagon is inscribed in a circle of radius 10. Find the perimeter of the pentagon.

(18) From a hot air balloon 2 km high, the angles of depression of two towns in line with the balloon and on the same side of the balloon are $81^\circ$ and $13^\circ$. How far apart are the towns (to the nearest km)?