Solutions to Review for Exam 3

MATH 1190 sec. 51

Sections Covered: 3.3 (Log. Diff), 4.5, 4.2, 4.3, 4.4, 4.8, 5.1, 5.2

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find all critical numbers of the function.

(a) $f(x) = x^3 - 6x \quad \pm\sqrt{2}$

(b) $f(x) = \dfrac{x}{x^2+1} \quad \pm 1$

(c) $f(x) = x^2\sqrt{3-x} \quad 0, 3, 12/5$

(d) $f(x) = (x-2)^3(x+4)^5 \quad 2, -4, -1/4$

(2) Find the absolute maximum and minimum values of the function and where they occur on the given interval.

(a) $f(x) = x^3 - 6x \quad [-1, 1]$

The absolute minimum is -5 taken 1. The absolute maximum is 5 taken at -1.

(b) $f(x) = \dfrac{x}{x^2+1} \quad [-2, 2]$

The absolute minimum is $-1/2$ taken at -1. The absolute maximum is $1/2$ taken at 1.

(3) Determine where the function is increasing, decreasing, concave up, concave down, and find the location (e.g. x-value) of all extrema and points of inflection.

(a) $y = x^2(2x^2 - 9)$

y is increasing on $(-3/2, 0) \cup (3/2, \infty)$, decreasing on $(-\infty, -3/2) \cup (0, 3/2)$, concave up on $(-\infty, -\sqrt{3}/2) \cup (\sqrt{3}/2, \infty)$ and concave down on $(-\sqrt{3}/2, \sqrt{3}/2)$. It has local minima at $x = \pm 3/2$, and a local maximum at $x = 0$. There are two points of inflection at $x = \pm\sqrt{3}/2$.

(b) $f(x) = \dfrac{x}{x^2+1}$

f is increasing on $(-1,1)$ and decreasing on $(-\infty,-1)\cup(1,\infty)$. f is concave up on $(-\sqrt{3},0)\cup(\sqrt{3},\infty)$ and concave down on $(-\infty,-\sqrt{3})\cup(0,\sqrt{3})$. f has a local minimum at -1, a local maximum at 1, and points of inflection occuring at $-\sqrt{3}$, 0, and $\sqrt{3}$.

(4) Use logarithmic differentiation to find $\frac{dy}{dx}$.

(a) $y=(\ln x)^{\sin x}$ $\quad y'=(\ln x)^{\sin x}\left(\cos x\ln(\ln x)+\frac{\sin x}{x\ln x}\right)$

(b) $y=\frac{x\sin x}{\sqrt{x^2+3}}$ $\quad \frac{dy}{dx}=\frac{x\sin x}{\sqrt{x^2+3}}\left(\frac{1}{x}+\cot x-\frac{x}{x^2+3}\right)$

(c) $y=\sqrt{\frac{x+1}{x-1}}$ $\quad y'=\sqrt{\frac{x+1}{x-1}}\left(\frac{1}{2(x+1)}-\frac{1}{2(x-1)}\right)$

(5) Determine all antiderivatives of the function.

(a) $f(x)=x(x-1)$, $\quad F(x)=\frac{x^3}{3}-\frac{x^2}{2}+C$

(b) $h(x)=\cos x+\csc^2 x$, $\quad H(x)=\sin x-\cot x+C$

(c) $y=4\sin x-3\sec x\tan x$, $\quad Y=-4\cos x-3\sec x+C$

(d) $f(x)=\frac{3x^3+2x^2+4}{x}$, $\quad F(x)=x^3+x^2+4\ln|x|+C$

(e) $h(x)=\frac{4}{\sqrt{1-x^2}}$, $\quad H(x)=4\sin^{-1}(x)+C$

(6) Evaluate each limit if it exists using any appropriate techniques.

(a) $\lim_{x\to 0}\frac{\tan(2x)}{\ln(1+x)}=2$

(b) $\lim_{x\to 1}\frac{\ln(x)}{e^{x-1}}=0$

(c) $\lim_{x\to 0}\frac{\ln(1-x)}{e^x-1}=-1$

(d) $\lim_{t\to 0}\frac{4^t-6^t}{t}=\ln 4-\ln 6$

(e) $\lim_{x \to \infty} \frac{x^3}{e^{2x}} = 0$

(f) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) = 1/2$

(g) $\lim_{x \to \infty} x^{1/\ln x} = e$

(7) Evaluate each integral by interpreting it in terms of areas.

(a) $\int_0^4 -\sqrt{16 - x^2}\, dx = -4\pi$

(b) $\int_0^5 (|x-1|-1)\, dx = \frac{7}{2}$

(c) $\int_0^2 f(x)\, dx = \frac{5}{2}$ where $f(x) = \begin{cases} 1, & x \leq 1 \\ x, & x > 1 \end{cases}$