Solutions to Review for Exam III
Calculus II sec. 001 Summer 2015

Sections Covered:7.3, 7.4, 7.5, 7.8, 8.1, 8.2, 8.3, 8.4, 8.5, 8.6

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Evaluate the given integrals using any applicable method.

(a) $\displaystyle\int \frac{\sqrt{x^2-9}}{x}\,dx = \sqrt{x^2-9}-3\sec^{-1}\frac{x}{3}+C$

(b) $\displaystyle\int \frac{dy}{\sqrt{16-y^2}} = \sin^{-1}\frac{y}{4}+C$

(c) $\displaystyle\int \frac{dx}{x^2-1} = \frac{1}{2}\ln|x-1|-\frac{1}{2}\ln|x+1|+C$

(d) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx = \frac{1}{3}(x^2+1)^{3/2}-\sqrt{x^2+1}+C$

(e) $\displaystyle\int \frac{x^2+7x+2}{(x^2+1)(x+3)}\,dx = -\ln|x+3|+\ln|x^2+1|+\tan^{-1}(x)+C$

(f) $\displaystyle\int_0^1 \sqrt{1-x^2}\,dx = \frac{\pi}{4}$

(2) Find the form of the partial fraction decomposition. (It is not necessary to find any of the coefficients A, B, etc.)

(a) $\displaystyle\frac{2x}{x^2+7x+12} = \frac{A}{x+3}+\frac{B}{x+4}$

(b) $\displaystyle\frac{x^2+2x-1}{(x^2-2x+1)(x^2-4)} = \frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+2}+\frac{D}{x-2}$

(c) $\displaystyle\frac{1}{(x+2)^3(x^2-1)^2(x^2+4)^3} = \frac{A}{x+2}+\frac{B}{(x+2)^2}+\frac{C}{(x+2)^3}+\frac{D}{x-1}+\frac{E}{(x-1)^2}+\frac{F}{x+1}+$

$$+\frac{G}{(x+1)^2}+\frac{Hx+I}{x^2+4}+\frac{Jx+K}{(x^2+4)^2}++\frac{Lx+M}{(x^2+4)^3}$$

(3) Write the following as the sum of a polynomial and a proper rational function. Find a partial fraction decomposition for the resulting proper rational function.

$$\frac{x^4+x^3+9x^2+8x-11}{(x+1)(x^2+9)} = x - \frac{1}{x+1} + \frac{x-2}{x^2+9}$$

(4) Evaluate the given integral or determine that it is divergent.

(a) $\displaystyle\int_{-\infty}^{2} \frac{1}{x^2+4}\,dx = \frac{3\pi}{8}$

(b) $\displaystyle\int_{0}^{2} \frac{1}{x-1}\,dx$ divergent

(c) $\displaystyle\int_{0}^{1} \ln\sqrt{y}\,dy = -\frac{1}{2}$

(5) Determine if the sequence converges or diverges. If it converges, find its limit.

(a) $a_n = \ln(2n^2+1) - \ln(n^2)$ $\quad L = \ln(2)$

(b) $b_n = 3 + 4(-1)^n$ divergent–it oscillates

(c) $c_n = \dfrac{3n^2+2n+1}{2-n^2}$ $\quad L = -3$

(6) Find the sum of the convergent telescoping series.

$$\sum_{k=1}^{\infty} \frac{1}{k^2+5k+6} = \frac{1}{3}$$

(7) Determine if the geometric series is convergent or divergent. If convergent, find its sum.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n}} = 9$

(b) $4 - \dfrac{1}{2} + \dfrac{1}{16} - \dfrac{1}{128} + \cdots = \dfrac{32}{9}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2^{5n+1}}{5^{2n-1}}$ divergent $r = \dfrac{32}{25}$

(8) Determine if the given series converges or diverges. If the series converges, determine if it converges absolutely or conditionally. Make clear what tests if any you are using and what your conclusion is in each case.

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$ absolutely convergent, integral test

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{2^{4n+1}}$ absolutely convergent, geometric

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}$ divergent, divergence test

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ conditionally convergent, alternating series test & direct comparison

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{5^n n!}$ absolutely convergent, ratio test

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{n+\sqrt{n+1}}{\sqrt[3]{n^5-4n^3+2n+3}}$ divergent, limit comparison test

(9) These are some conceptual questions.

(a) If $\lim_{n\to\infty} a_n = 0$, then the sequence $\{a_n\}$ is convergent. (T/F) **T**

(b) If $\lim_{n\to\infty} a_n = 0$, then the series $\sum a_n$ must be convergent. (T/F) **F**

(c) If $\{a_n\}$ is a sequence of real numbers, then it has a derivative $\{a_n'\}$. (T/F) **F**

(d) If $\lim_{n\to\infty} a_n = 2$, then the series $\sum a_n$ must be divergent. (T/F) **T**

(e) If the series of positive terms $\sum a_n$ is convergent, it must be absolutely convergent. (T/F) **T**

(f) The integral test may be used to determine if $\sum_{n=1}^{\infty} \frac{n!}{2^n}$ is convergent or divergent. (T/F) **F**

(g) The terms *sequence* and *series* are interchangeable. (T/F) **F**

(h) If the sequence $\{a_n\}$ converges, then the series $\sum a_n$ converges. (T/F) **F**