Review for Exam III

MATH 2306 sections 51 & 54

Sections Covered: 4.6, 4.9, 5.1, 7.1, 7.2

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) A 1 kg mass is attached to a spring whose spring constant is 13 N/m. The surrounding medium offers a resistance that is numerically equal to 6 times the instantaneous velocity. The mass is released from rest 2 m above the equilibrium position. Determine the equation of motion.

(2) An LRC series circuit exhibits free electrical vibrations. If the inductance $L = 1h$ and the capacitance $C = 0.04f$, determine the resistance R $(R > 0)$ such that the electrical vibrations are **(a)** overdamped, **(b)** underdamped, and **(c)** critically damped.

(3) Find the Laplace transform using the definition and specify its domain.

$$f(t) = te^{2t}.$$

(4) Find the Laplace transform using any method.

(a) $f(t) = (t-1)^2 - e^{-3t}$

(b) $f(t) = t + \sin \pi t$

(c) $f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 1, & 1 \leq t \end{cases}$

(5) Find the inverse Laplace transform using any method.

(a) $F(s) = \dfrac{1}{s^2 - 25}$

(b) $F(s) = \dfrac{2s+5}{s^3+3s}$

(c) $F(s) = \dfrac{4}{s(s+1)}$

(6) Solve the IVP using the Laplace transform.

(a) $y''+4y = 1 \quad y(0) = 0, \quad y'(0) = -1$

(b) $y''-y = 2\cos(5t) \quad y(0) = 0, \quad y'(0) = 0$

(7) Find $\mathscr{L}\{f(t)\}$ given that

$$\mathscr{L}\{f'(t)\} = \ln\left(\frac{s^2+4}{s^2}\right) \quad \text{and} \quad f(0) = 1$$

Don't waste any time or energy trying to find f. You don't need to.

(8) Find the general solution of the ODE.

$y''+y = \csc x$

(9) Find the general solution of the ODE for which one solution to the associated homogeneous equation, y_1, is given.

$$x^2y'' + 3xy' - 3y = 15x^2, \quad y_1(x) = x^{-3}$$

(10) A 160 pound weight is attached to an industrial spring causing it to stretch from 9 feet to 10.28 feet. The weight is then driven from rest at equilibrium by an external for $F(t) =$

$F_0 \cos(\gamma t)$. At what frequency γ will the external force induce pure resonance? (Take $g = 32$ feet per second squared.)

(11) Solve the IVP.

$$\begin{aligned} \frac{dx}{dt} &= y \qquad x(0) = 1 \\ \frac{dy}{dt} &= x \qquad y(0) = 0 \end{aligned}$$

Note that the hyperbolic sine and cosine provide a nice, compact formulation of the solution.