Solutions to Review for Exam III

MATH 2306 (Ritter)

Sections Covered: 8, 9

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the general solution of the homogeneous equation.

(a) $y''-2y'+5y=0$ $\quad y=c_1e^x\cos(2x)+c_2e^x\sin(2x)$

(b) $y''+6y'+9y=0$ $\quad y=c_2e^{-3x}+c_2xe^{-3x}$

(c) $y''-36y=0$ $\quad y=c_1e^{6x}+c_2e^{-6x}$

(d) $y^{(4)}+3y''-4y=0$ $\quad y=c_1\cos(2x)+c_2\sin(2x)+c_3e^x+c_4e^{-x}$

(e) $y'''+2y''+y'=0$ $\quad y=c_1+c_2e^{-x}+c_3xe^{-x}$

(f) $2y''-3y'-2y=0$ $\quad y=c_1e^{-x/2}+c_2e^{2x}$

(2) Solve each IVP

(a) $y''-3y'+2y=0 \quad y(0)=0, \quad y'(0)=2$ $\quad y=2e^{2x}-2e^x$

(b) $y''+2y'=0 \quad y(1)=0, \quad y'(1)=1$ $\quad y=\frac{1}{2}-\frac{e^2}{2}e^{-2x}$

(c) $y''-2y'+5y=0 \quad y(0)=0, \quad y'(0)=2$ $\quad y=e^x\sin(2x)$

(4) Find the general solution of each nonhomogeneous equation

(a) $y''+6y'+9y=e^x+3e^{-3x}$ $\quad y=c_1e^{-3x}+c_2xe^{-3x}+\frac{1}{16}e^x+\frac{3}{2}x^2e^{-3x}$

(b) $y''+y'-12y=2x$ $\quad y=c_1e^{-4x}+c_2e^{3x}-\frac{1}{6}x-\frac{1}{72}$

(c) $y''+y=4\cos x \qquad y=c_1\cos x+c_2\sin x+2x\sin x$

(5) Determine the **form** of the particular solution. (Do not bother trying to find any of the coefficients A, B, etc.)

(a) $y''-4y'+5y=x\cos 2x \qquad y_p=(Ax+B)\cos(2x)+(Cx+D)\sin(2x)$

(b) $y''+y=x^3+e^x \qquad y_p=Ax^3+Bx^2+Cx+D+Ee^x$

(c) $y''-4y'+5y=xe^{2x}\sin x \qquad y_p=(Ax^2+Bx)e^{2x}\sin x+(Cx^2+Dx)e^{2x}\cos x$

(d) $y''-2y'+y=1+e^x \qquad y_p=A+Bx^2e^x$

(6) For each homogeneous equation, write out the characteristic equation. If the equation doesn't have a characteristic equation, briefly state why.

(a) $3\dfrac{d^4y}{dx^4}-2\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}-4y=0 \qquad 3m^4-2m^3+m-4=0$

(b) $4y''+2xy'+e^xy=0$ none exists, it's not constant coefficient

(c) $x^3y'''+2x^2y''-4xy'+y=0$ none exists, it's not constant coefficient

(d) $y^{(6)}+16y^{(4)}-12y''+y=0 \qquad m^6+16m^4-12m^2+1=0$

(7) For each of the following nonhomogeneous equations, determine whether the method of undetermined coefficients **could** be used to determine y_p. If not, give a brief explanation.

(a) $3\dfrac{d^4y}{dx^4}-2\dfrac{d^3y}{dx^3}+\dfrac{dy}{dx}-4y=x^3e^x$ yes, it could

(b) $4y''+2y'+y=\dfrac{1}{1+x^2}$ nope, RHS is not of the correct type

(c) $x^3y'''+2x^2y''-4xy'+y=\sin(2x)+x$ nope, LHS is not constant coefficient

(d) $y^{(6)}+16y^{(4)}-12y''+y=x\ln x$ nope, RHS is not of the correct type