Review for Exam 3

MATH 2306 (Ritter)

Sections Covered: 9, 10, 11, 12, 13, 14

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the general solution of each nonhomogeneous equation

(a) $y''+6y'+9y = e^{x}+3e^{-3x}$

(b) $y''+y'-12y = 2x$

(c) $y''+y = 4\cos x$

(2) Determine the **form** of the particular solution. (Do not bother trying to find any of the coefficients A, B, etc.)

(a) $y''-4y'+5y = x\cos 2x$

(b) $y''+y = x^3+e^x$

(c) $y''-4y'+5y = xe^{2x}\sin x$

(d) $y''-2y'+y = 1+e^x$

(3) A 64 lb object is attached to a spring whose spring constant is 26 lb/ft. A dashpot provides damping that is numerically equal to 8 times the instantaneous velocity.

(a) Determine the mass m of the object in slugs.

(b) Assuming there is no external applied force, set up the differential equation for the displacement and determine if the motion is overdamped, underdamped or critically damped.

(c) If the object is initially displaced 6 inches above equilibrium and given an initial upward velocity of 2 ft/sec, determine the displacement for $t > 0$.

(4) A 200 volt battery is applied to a series circuit with inductance 2 henries, resistance 26 ohms and capacitance $\frac{1}{80}$ farads. Find the charge on the capacitor $q(t)$ for $t > 0$ assuming the initial charge and current are zero, $q(0) = 0$, $i(0) = 0$.

(5) A 2 slug object is attached to a spring whose spring constant is 162 lb/ft. The system is undamped, and an external driving force of $f(t) = -4\cos(\gamma t)$ is applied. Assume that the driving force induces pure resonance. If the object starts from rest at equilibrium, determine the displacement for $t > 0$. If the spring has a maximum stretched length of 4 ft, after how many seconds will the amplitude of the oscillations exceed the maximum spring length?

(6) Consider the nonhomogeneous equation $x^2y'' + xy' - 4y = 20x^3$.

(a) One solution of the associated homogeneous equation is $y_1 = x^2$. Find a second linearly independent one y_2.

(b) Find a particular solution y_p of the nonhomogeneous equation.

(c) Solve the IVP: $x^2y'' + xy' - 4y = 20x^3,\ y(1) = 3,\ y'(1) = 6$.

(7) Use the method of variation of parameters to find a particular solution for each nonhomogeneous equation.

(a) $y''+y = \sec\theta\tan\theta, \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

(b) $y''+3y'+2y = \sin(e^x)$

(8) Use the definition (i.e. compute an integral) to show that $\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$ for $s > a$.

(9) Compute the transform or inverse transform as indicated. (Use the table of Laplace transforms along with any necessary algebra or identities.)

(a) $\mathscr{L}\{(2t-3)^2\}$

(b) $\mathscr{L}\{\cos^2 t - \sin^2 t\}$ (hint: double angle formula)

(c) $\mathscr{L}\{e^{3t} + 7\sin(2t) - t^5\}$

(d) $\mathscr{L}^{-1}\left\{\dfrac{1}{s^4} - \dfrac{s}{s^2+5}\right\}$

(e) $\mathscr{L}^{-1}\left\{\dfrac{5s+3}{s^2+s}\right\}$