Solutions to Review for Exam 3

MATH 2306 (Ritter)

Sections Covered: 9, 10, 11, 12, 13, 14

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the general solution of each nonhomogeneous equation

(a) $y''+6y'+9y = e^x+3e^{-3x}$ $y = c_1 e^{-3x} + c_2 x e^{-3x} + \dfrac{1}{16}e^x + \dfrac{3}{2}x^2 e^{-3x}$

(b) $y''+y'-12y = 2x$ $y = c_1 e^{-4x} + c_2 e^{3x} - \dfrac{1}{6}x - \dfrac{1}{72}$

(c) $y''+y = 4\cos x$ $y = c_1 \cos x + c_2 \sin x + 2x \sin x$

(2) Determine the **form** of the particular solution. (Do not bother trying to find any of the coefficients A, B, etc.)

(a) $y''-4y'+5y = x\cos 2x$ $y_p = (Ax + B)\cos(2x) + (Cx + D)\sin(2x)$

(b) $y''+y = x^3+e^x$ $y_p = Ax^3 + Bx^2 + Cx + D + Ee^x$

(c) $y''-4y'+5y = xe^{2x}\sin x$ $y_p = (Ax^2 + Bx)e^{2x}\sin x + (Cx^2 + Dx)e^{2x}\cos x$

(d) $y''-2y'+y = 1+e^x$ $y_p = A + Bx^2 e^x$

(3) A 64 lb object is attached to a spring whose spring constant is 26 lb/ft. A dashpot provides damping that is numerically equal to 8 times the instantaneous velocity.

(a) Determine the mass m of the object in slugs. $m = 2$ **slugs**

(b) Assuming there is no external applied force, set up the differential equation for the displacement and determine if the motion is overdamped, underdamped or critically damped.

$2x'' + 8x' + 26x = 0$, i.e. $x'' + 4x' + 13x = 0$. The system is underdamped with characteristic roots $r = -2 \pm 3i$.

(c) If the object is initially displaced 6 inches above equilibrium and given an initial upward velocity of 2 ft/sec, determine the displacement for $t > 0$. $x(t) = \frac{1}{2}e^{-2t}\cos(3t) + e^{-2t}\sin(3t)$

(4) A 200 volt battery is applied to a series circuit with inductance 2 henries, resistance 26 ohms and capacitance $\frac{1}{80}$ farads. Find the charge on the capacitor $q(t)$ for $t > 0$ assuming the initial charge and current are zero, $q(0) = 0$, $i(0) = 0$. $q(t) = \frac{25}{6}e^{-8t} - \frac{20}{3}e^{-5t} + \frac{5}{2}$

(5) A 2 slug object is attached to a spring whose spring constant is 162 lb/ft. The system is undamped, and an external driving force of $f(t) = -4\cos(\gamma t)$ is applied. Assume that the driving force induces pure resonance. If the object starts from rest at equilibrium, determine the displacement for $t > 0$. If the spring has a maximum stretched length of 4 ft, after how many seconds will the amplitude of the oscillations exceed the maximum spring length? Resonance frequency is $\omega = \sqrt{162/2} = 9$ per second. The IVP is $x'' + 81x = -2\cos(9t)$ with $x(0) = x'(0) = 0$. The displacement is $x(t) = -\frac{1}{9}t\sin(9t)$. The amplitude $|t/9|$ will exceed 4 ft when $t > 36$ seconds. So after 36.

(6) Consider the nonhomogeneous equation $x^2 y'' + xy' - 4y = 20x^3$.

(a) One solution of the associated homogeneous equation is $y_1 = x^2$. Find a second linearly independent one y_2. $y_2 = \frac{1}{x^2}$

(b) Find a particular solution y_p of the nonhomogeneous equation. $y_p = 4x^3$

(c) Solve the IVP: $x^2 y'' + xy' - 4y = 20x^3$, $y(1) = 3$, $y'(1) = 6$. $y = -2x^2 + \frac{1}{x^2} + 4x^3$

(7) Use the method of variation of parameters to find a particular solution for each nonhomogeneous equation.

(a) $y'' + y = \sec\theta\tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2} \quad y_p = \theta\cos\theta + \sin\theta\ln(\sec\theta)$

$y_p = (\theta - \tan \theta) \cos \theta + \sin \theta \ln(\sec \theta)$ is also correct.

(b) $y'' + 3y' + 2y = \sin(e^x)$ $y_p = -e^{-2x} \sin(e^x)$

(8) Use the definition (i.e. compute an integral) to show that $\mathscr{L}\{e^{at}\} = \frac{1}{s-a}$ for $s > a$. Use the

fact that $e^{-st}e^{at} = e^{-(s-a)t}$. The convergence of the integral will require $s - a > 0$.

(9) Compute the transform or inverse transform as indicated. (Use the table of Laplace trans-

forms along with any necessary algebra or identities.)

(a) $\mathscr{L}\{(2t-3)^2\} = \mathscr{L}\{4t^2 - 12t + 9\} = \dfrac{8}{s^3} - \dfrac{12}{s^2} + \dfrac{9}{s}$

(b) $\mathscr{L}\{\cos^2 t - \sin^2 t\} = \mathscr{L}\{\cos(2t)\} = \dfrac{s}{s^2+4}$

(c) $\mathscr{L}\{e^{3t} + 7\sin(2t) - t^5\} = \dfrac{1}{s-3} + \dfrac{14}{s^2+4} - \dfrac{120}{s^6}$

(d) $\mathscr{L}^{-1}\left\{\dfrac{1}{s^4} - \dfrac{s}{s^2+5}\right\} = \mathscr{L}^{-1}\left\{\dfrac{1}{6}\dfrac{3!}{s^4} - \dfrac{s}{s^2+(\sqrt{5})^2}\right\} = \dfrac{1}{6}t^3 - \cos(\sqrt{5}t)$

(e) $\mathscr{L}^{-1}\left\{\dfrac{5s+3}{s^2+s}\right\} = \mathscr{L}^{-1}\left\{\dfrac{3}{s} - \dfrac{2}{s+1}\right\} = 3 + 2e^{-t}$