Practice for Exam 3 MATH 3260 Fall 2017

Sections Covered: 6.1, 6.2, 6.3, 6.4, 6.7, 5.1, 5.2

This practice exam is intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) Suppose A is a square matrix and 2 is an eigenvalue of A. Explain why the matrix $A - 2I$ is a singular matrix.

(2) Determine the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(3) For each eigenvalue of the matrix in the previous problem find a basis for the corresponing eigenspace.

(4) Determine all real or complex eigenvalues of the matrix $\begin{bmatrix} 7 & -5 \\ 1 & 3 \end{bmatrix}$.

(5) Find a unit vector in the direction of $\mathbf{v} = (1, 2, 1, -3)$.

(6) Write the vector $\mathbf{u} = (0, -2, 3, 2)$ in the form $\mathbf{u} = \hat{\mathbf{u}} + \mathbf{z}$ where $\hat{\mathbf{u}}$ is parallel to $\mathbf{v}$ and $\mathbf{z}$ is orthogonal to $\mathbf{v}$ for the vector $\mathbf{v}$ in problem (5). Use this to find the distance between the point $(0, -2, 3, 2)$ and the line Span$\{\mathbf{v}\}$ in $\mathbb{R}^4$.

(7) Find a basis for the orthogonal complement $[\,\text{Row}(A)]^{\perp}$ of the row space of the given matrix.

$$A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$$

(8) Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(9) Define and inner product on $\mathbb{R}^2$ by $\langle \mathbf{u}, \mathbf{v} \rangle = \mathbf{v}^T A \mathbf{u}$ where $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$. Relative to this inner product

(a) Find the norm of $(1, 1)$.

(b) Find a unit vector in the direction of $(1, 1)$.

(c) Characterize the set of all vectors that are orthogonal to $(1, 2)$.

(d) Find the distance between $(1, 1)$ and $(1, 2)$.

(10) Find the least squares best fit line to the data

x	0	1	2	3	4	5
y	-1	1	1	3	4	6

(11) For the product $\langle \mathbf{p}, \mathbf{q} \rangle = \mathbf{p}(-1)\mathbf{q}(-1) + \mathbf{p}(0)\mathbf{q}(0) + \mathbf{p}(1)\mathbf{q}(1)$ on $\mathbb{P}_2$, show that for each $\mathbf{p}$

$$\langle \mathbf{p}, \mathbf{p} \rangle \geq 0, \quad \text{and} \quad \langle \mathbf{p}, \mathbf{p} \rangle = 0$$

if and only if $\mathbf{p} = \mathbf{0}$.