Practice for Exam 3 (Ritter) MATH 3260 Spring 2018

Sections Covered: 4.4, 4.5, 4.6, 6.1, 6.2, 6.3, 6.4, 5.1, 5.2

These practice problems are intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

(1) For each matrix A, find bases for Nul$A$, Col$A$, and Row$A$. Determine both rank$A$ and dim(Nul$A$).

$$\text{(a)}\quad A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}, \quad \text{(b)}\quad A = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}$$

(2) Show that the given set is a basis for $\mathbb{R}^2$. Determine the change of coordinates matrix $P_{\mathcal{B}}$ and its inverse (use the order presented here). Then use this to find the indicated coordinate vectors.

$$\mathcal{B} = \left\{ \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

Determine $[\mathbf{x}]_{\mathcal{B}}$ for

$$\text{(a)}\quad \mathbf{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad \text{(b)}\quad \mathbf{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \text{(c)}\quad \mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

(3) Determine the dimension of the indicated vector space.

(a) The subspace of $\mathbb{R}^4$ of vectors whose components sum to zero.

(b) The subspace of $\mathbb{P}_4$ consisting of polynomials of the form $\mathbf{p}(t) = at^4 + b(t^2 - t)$, for real numbers a and b.

(c) The null space of a 5×8 matrix with a rank of 4.

(d) The row space of an $m \times n$ matrix whose null space has dimension 6. (It may be necessary to express the answer in terms of m or n or both.)

(4) Find a unit vector in the direction of $\mathbf{v} = (1, 2, 1, -3)$.

(5) Write the vector $\mathbf{u} = (0, -2, 3, 2)$ in the form $\mathbf{u} = \hat{\mathbf{u}} + \mathbf{z}$ where $\hat{\mathbf{u}}$ is parallel to $\mathbf{v}$ and $\mathbf{z}$ is orthogonal to $\mathbf{v}$ for the vector $\mathbf{v}$ in problem (4). Use this to find the distance between the point $(0, -2, 3, 2)$ and the line Span$\{\mathbf{v}\}$ in $\mathbb{R}^4$.

(6) Find a basis for $[\ \text{Row}(A)]^\perp$, the orthogonal complement of the row space of the given matrix.

$$A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$$

(7) Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

(8) Suppose A is a square matrix and 2 is an eigenvalue of A. Explain why the matrix $A - 2I$ is a singular matrix.

(9) Determine the eigenvalues of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

(10) For each eigenvalue of the matrix in the previous problem find a basis for the corresponing eigenspace.