Practice for Exam 3 (Ritter) MATH 3260 Spring 2020

Sections Covered: 4.4, 4.5, 4.6, 6.1, 6.2, 6.3, 6.4
These practice problems are intended to give you a rough idea of the types of problems you can expect to encounter. **Nothing else is intended or implied.**

1. For each matrix A, find bases for Nul$A$, Col$A$, and Row$A$. Determine both rank$A$ and dim(Nul$A$).

$$\text{(a)} \quad A = \begin{bmatrix} 1 & 3 & 4 & -1 & 2 \\ 2 & 6 & 6 & 0 & -3 \\ 3 & 9 & 3 & 6 & -3 \\ 3 & 9 & 0 & 9 & 0 \end{bmatrix}, \quad \text{(b)} \quad A = \begin{bmatrix} 1 & 1 & -2 & 0 & 1 & -2 \\ 1 & 2 & -3 & 0 & -2 & -3 \\ 1 & -1 & 0 & 0 & 1 & 6 \\ 1 & -2 & 2 & 1 & -3 & 0 \\ 1 & -2 & 1 & 0 & 2 & -1 \end{bmatrix}$$

2. Show that the given set is a basis for $\mathbb{R}^2$. Determine the change of coordinates matrix $P_{\mathcal{B}}$ and its inverse (use the order presented here). Then use this to find the indicated coordinate vectors.

$$\mathcal{B} = \left\{ \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

Determine $[\mathbf{x}]_{\mathcal{B}}$ for

$$\text{(a)} \quad \mathbf{x} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}, \quad \text{(b)} \quad \mathbf{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \text{(c)} \quad \mathbf{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

3. Determine the dimension of the indicated vector space.

(a) The subspace of $\mathbb{R}^4$ of vectors whose components sum to zero.

(b) The subspace of $\mathbb{P}_4$ consisting of polynomials of the form $\mathbf{p}(t) = at^4 + b(t^2 - t)$, for real numbers a and b.

(c) The null space of a 5×8 matrix with a rank of 4.

(d) The row space of an $m \times n$ matrix whose null space has dimension 6. (It may be necessary to express the answer in terms of m or n or both.)

(e) The subspace of $M^{3\times 3}$ consisting of matrices in which the entries in each column sum to zero (e.g. $[a_{ij}]$ such that $a_{1j} + a_{2j} + a_{3j} = 0$ for each $j = 1, 2, 3$.)

4. Find a unit vector in the direction of $\mathbf{v} = (1, 2, 1, -3)$.

5. Write the vector $\mathbf{u} = (0, -2, 3, 2)$ in the form $\mathbf{u} = \hat{\mathbf{u}} + \mathbf{z}$ where $\hat{\mathbf{u}}$ is parallel to $\mathbf{v}$ and $\mathbf{z}$ is orthogonal to $\mathbf{v}$ for the vector $\mathbf{v}$ in problem (4). Use this to find the distance between the point $(0, -2, 3, 2)$ and the line Span$\{\mathbf{v}\}$ in $\mathbb{R}^4$.

6. Find a basis for $[\,\text{Row}(A)]^\perp$, the orthogonal complement of the row space of the given matrix.

$$A = \begin{bmatrix} 5 & 1 & 2 & 2 & 0 \\ 3 & 3 & 2 & -1 & -12 \\ 8 & 4 & 4 & -5 & 12 \\ 2 & 1 & 1 & 0 & -2 \end{bmatrix}$$

7. Find an orthonormal basis for the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

8. Consider the set $\{\mathbf{u}_1, \mathbf{u}_2\} = \left\{ \begin{bmatrix} 2 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} -6 \\ 2 \\ 0 \end{bmatrix} \right\}$. Find the orthogonal projection of $\mathbf{y} = \begin{bmatrix} 3 \\ 5 \\ -4 \end{bmatrix}$ onto Span$\{\mathbf{u}_1, \mathbf{u}_2\}$, and find the distance between $\mathbf{y}$ and the plane Span$\{\mathbf{u}_1, \mathbf{u}_2\}$.

9. The first four Chebyshev polynomials are

$$T_0(t) = 1, \quad T_1(t) = t, \quad T_2(t) = 2t^2 - 1, \quad T_3(t) = 4t^3 - 3t.$$

(a) Show that the set $\{T_0, T_1, T_2, T_3\}$ is linearly independent in $\mathbb{P}^3$.

(b) Let $\mathbf{p}(t) = 1 + t + t^2 + t^3$. Find the coordinate vector $[\mathbf{p}]_\mathcal{C}$ relative to the basis $\mathcal{C} = \{T_0, T_1, T_2, T_3\}$.

(c) Find the polynomial $\mathbf{q}$ in $\mathbb{P}_3$ whose coordinate vector $[\mathbf{q}]_{\mathcal{C}} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

10. Suppose a nonhomogeneous system of six linear equations in eight unknowns has a solution, with two free variables. Is it possible to change some constants on the equations' right sides to make the new system inconsistent? (Why/why not?)

11. Let $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$. Find two nonzero, non-parallel vectors in $W^{\perp}$.