

Review for Exam IV

MATH 1112 Section 54 Spring 2019

Sections Covered: 7.4, 7.1, 7.2, 7.3, 7.5

Calculator Policy: Calculator use may be allowed on part of the exam. When instructions call for an **exact** solution, that indicates that a decimal approximation will not be accepted.

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Consider each trigonometric equation. Determine if the equation is an identity or if it is conditional (i.e. not an identity). If it is an identity, prove it. If it is not an identity, find at least one value of the variable for which the equation is not satisfied.

(a) $\frac{\sin x}{1 - \cos x} = \frac{1 + \cos x}{\sin x}$

(b) $(\cos \theta + \sin \theta)^2 = 1$

(c) $\sin^2 \beta \tan^2 \beta = \tan^2 \beta - \sin^2 \beta$

(d) $\frac{\tan x + \cot x}{\csc x} = \sec x$

(e) $\cos^2 t + \cot^2 t = \sin t \sec t$

(2) Given $\cos(u - v) = \cos u \cos v + \sin u \sin v$ and $\sin(u - v) = \sin u \cos v - \sin v \cos u$, obtain expressions for each of the following.

(a) $\cos(u + v)$

(b) $\sin(u + v)$

(c) $\cos(2u)$

(d) $\sin(2u)$

(3) Suppose $\sin \alpha = \frac{1}{3}$ and $\cos \beta = -\frac{1}{5}$. Further suppose that $\frac{\pi}{2} < \alpha < \pi$ and $\pi < \beta < \frac{3\pi}{2}$.

Evaluate each of the following exactly.

(a) $\cos(\alpha - \beta)$

(b) $\sin(2\beta)$

(c) $\sin(\alpha + \beta)$

(4) Find an equivalent algebraic expression for each of the following (*algebraic* meaning without any trigonometric functions).

(a) $\sin(\sin^{-1}(2x))$

(b) $\cos(\sin^{-1}(2x))$

(c) $\sec(\tan^{-1} u)$

(5) Use the sum of angles formula for the cosine that you obtained in (2)(a) to show that if $f(x) = \cos x$, then

$$\frac{f(x+h) - f(x)}{h} = \cos x \left(\frac{\cos h - 1}{h} \right) - \sin x \left(\frac{\sin h}{h} \right)$$

(6) Evaluate each expression exactly.

(a) $\cos(\ln(1))$

(b) $\tan^{-1}(e^0)$

(c) $e^{\tan 0^\circ}$

(d) $e^{\cos 90^\circ}$

(e) $\sin^{-1}(\ln e)$

(f) $\cos^{-1}(\ln 1)$

(7) Find all solutions of the given trigonometric equation on the interval $[0, 2\pi)$.

(a) $2 \sin x - \sqrt{3} = 0$

(b) $\sin(2x) + \cos x = 0$

(c) $\tan^2 x = 1$

(d) $2 \cos^2 x - 1 = \cos x$

(e) $\sin x + 1 = 2 \cos^2 x$

(8) Evaluate each expression exactly. No calculator is needed for these.

(a) $\cos(75^\circ)$

(b) $\cos(80^\circ) \cos(20^\circ) + \sin(80^\circ) \sin(20^\circ)$

(c) $\sin\left(\frac{\pi}{12}\right)$

(d) $\sin(12^\circ) \cos(57^\circ) - \sin(57^\circ) \cos(12^\circ)$

(e) $\cos\left(\frac{7\pi}{12}\right)$

(f) $\tan\left(\frac{11\pi}{12}\right)$

The following problem is from sections 8.1 and 8.2. It will not be on this fourth exam, but it will be on the final exam. So this review can be helpful in preparing for the final after the fourth test.

(9) Solve each triangle given the information provided. (We use the standard convention that the angles A, B, C are opposite sides of length $a, b,$ and $c,$ respectively. Express side lengths to the nearest hundredth and angles to the nearest degree.

(a) $C = 140^\circ, b = 1, c = 9$

(b) $A = 35^\circ, a = 6, b = 8$

(c) $a = 2, b = 3, C = 60^\circ$

(d) $a = 4, b = 3, c = 6$