Review for Exam IV

MATH 1113 sections 51 & 52 Fall 2018

Sections Covered: 6.3, 6.4, 6.5, 6.6, 7.4, 7.1, 7.2, 7.3, 7.5

Calculator Policy: Calculator use may be allowed on part of the exam. When instructions call for an **exact** solution, that indicates that a decimal approximation will not be accepted.

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Evaluate each trigonometric expression exactly if it exists. (Check with a calculator, but be able to do this without one. You can be sure I will ask you to do so on an exam.)

(a) $\cos\left(\frac{3\pi}{2}\right)$ (b) $\cot(2\pi)$ (c) $\csc\left(\frac{5\pi}{6}\right)$

(d) $\sin\left(\frac{11\pi}{6}\right)$ (e) $\tan\left(\frac{3\pi}{4}\right)$ (f) $\cos\left(\frac{5\pi}{4}\right)$

(g) $\sec\left(\frac{5\pi}{2}\right)$ (h) $\sec\left(\frac{2\pi}{3}\right)$ (i) $\tan\left(\frac{5\pi}{3}\right)$

(2) State the domain and range of each of the six trigonometric functions. Use interval notation or set builder notation.

(3) Identify the amplitude and period of each of each function.

(a) $f(x) = -3\cos\left(\frac{x}{2}\right) - 2$ (b) $g(x) = 4 - 4\sin\left(\pi x + \frac{\pi}{6}\right)$ (c) $F(x) = 4\sin\left(\frac{\pi}{4} - 2x\right)$

(4) Consider each trigonometric equation. Determine if the equation is an identity or if it is conditional (i.e. not an identity). If it is an identity, prove it. If it is not an identity, find at least one value of the variable for which the equation is not satisfied.

(a) $\dfrac{\sin x}{1-\cos x} = \dfrac{1+\cos x}{\sin x}$

(b) $(\cos\theta + \sin\theta)^2 = 1$

(c) $\sin^2\beta\tan^2\beta = \tan^2\beta - \sin^2\beta$

(d) $\dfrac{\tan x + \cot x}{\csc x} = \sec x$

(e) $\cos^2 t + \cot^2 t = \sin t \sec t$

(5) Given $\cos(u-v) = \cos u \cos v + \sin u \sin v$ and $\sin(u-v) = \sin u \cos v - \sin v \cos u$, obtain expressions for each of the following.

(a) $\cos(u+v)$

(b) $\sin(u+v)$

(c) $\cos(2u)$

(d) $\sin(2u)$

(6) Suppose $\sin\alpha = \frac{1}{3}$ and $\cos\beta = -\frac{1}{5}$. Futher suppose that $\frac{\pi}{2} < \alpha < \pi$ and $\pi < \beta < \frac{3\pi}{2}$. Evaluate each of the following exactly.

(a) $\cos(\alpha - \beta)$

(b) $\sin(2\beta)$

(c) $\sin(\alpha + \beta)$

(7) State the domain and the range of each of $f(x) = \sin^{-1}(x)$, $g(x) = \cos^{-1}(x)$ and $H(x) = \tan^{-1}(x)$ using interval notation.

(8) Evaluate each expression exactly if it exists. If it doesn't exist, state why.

(a) $\sin\left(\sin^{-1} 0.02\right)$

(b) $\sin^{-1}\left(\sin 0.02\right)$

(c) $\sin^{-1}\left[\sin\left(\pi\right)\right]$

(d) $\cos^{-1}\left[\cos\left(-\frac{\pi}{4}\right)\right]$

(e) $\cos\left(\tan^{-1} 4\right)$

(f) $\csc\left[\cos^{-1}\left(\frac{2}{3}\right)\right]$

(9) Find an equivalent algebraic expression for each of the following (*algebraic* meaning without any trigonometric functions).

(a) $\sin\left(\sin^{-1}(2x)\right)$

(b) $\cos\left(\sin^{-1}(2x)\right)$

(c) $\sec\left(\tan^{-1} u\right)$

(10) Use the sum of angles formula for the cosine that you obtained in (5)(a) to show that if $f(x) = \cos x$, then

$$\frac{f(x+h) - f(x)}{h} = \cos x \left(\frac{\cos h - 1}{h}\right) - \sin x \left(\frac{\sin h}{h}\right)$$

(11) Evaluate each expression exactly.

(a) $\cos\left(\ln(1)\right)$ (b) $\tan^{-1}\left(e^{0}\right)$ (c) $e^{\tan 0°}$

(d) $e^{\cos 90°}$ (e) $\sin^{-1}\left(\ln e\right)$ (f) $\cos^{-1}\left(\ln 1\right)$

(12) Find all solutions of the given trigonometric equation on the interval $[0, 2\pi)$.

(a) $2\sin x - \sqrt{3} = 0$

(b) $\sin(2x) + \cos x = 0$

(c) $\tan^2 x = 1$

(d) $2\cos^2 x - 1 = \cos x$

(e) $\sin x + 1 = 2\cos^2 x$

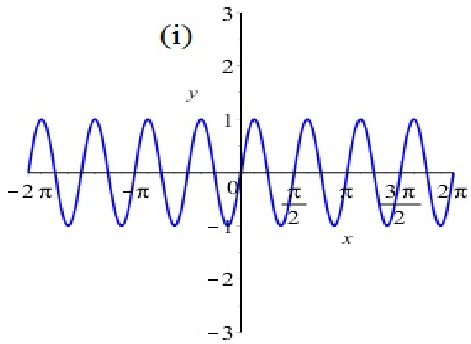
(13) Plot at least two full periods of each of $y = \sin x$, $y = \cos x$, and $y = \tan x$.

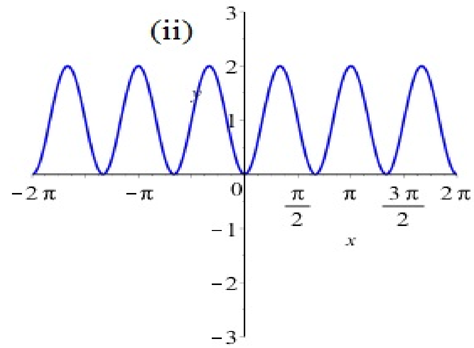
(14) Match the following functions with the plots shown. Note that not all of the functions will be used.

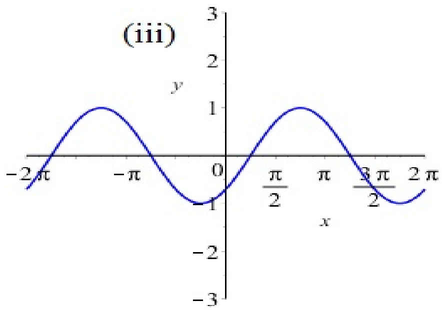
(a) $f(x) = 2 - \cos\left(x + \frac{\pi}{4}\right)$ (b) $f(x) = \sin(4x)$ (c) $f(x) = -2\sin(2x) + 1$

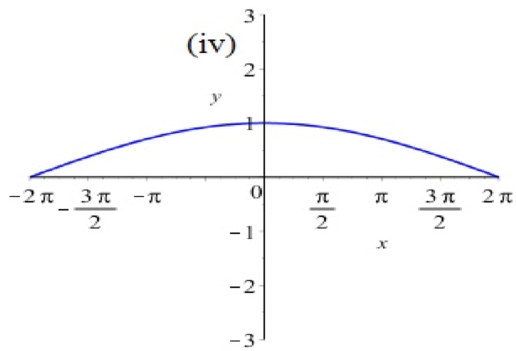
(d) $f(x) = -3\cos x + 1$ (e) $f(x) = -\cos(3x) + 1$ (f) $f(x) = \frac{1}{2}\sin(2x) - 2$

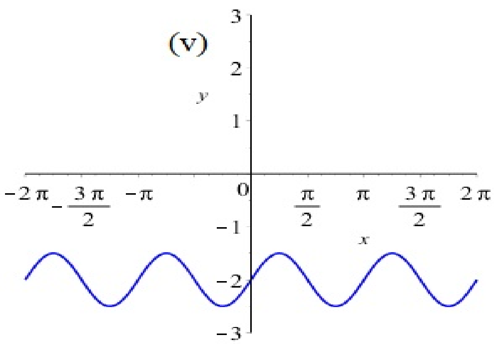
(g) $f(x) = 2 + \cos\left(\frac{\pi x}{4} - \frac{\pi}{2}\right)$ (h) $f(x) = \cos\left(\frac{x}{4}\right)$ (i) $f(x) = \sin\left(x - \frac{\pi}{4}\right)$

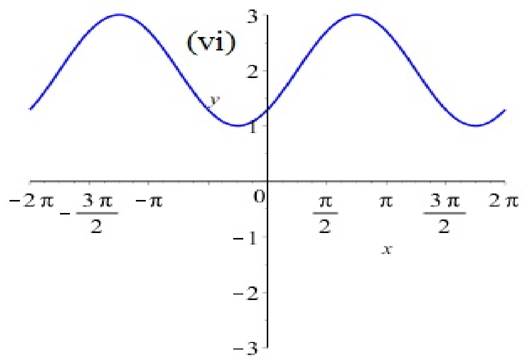












The following problem types would have to appear on a part of the exam in which calculator use is allowed.

(15) The hour hand on a certain clock is 4 inches long. Determine the length of the arc traversed by the tip of the hour hand between 1 pm and 6 pm (on the same day, so in five hours).

(16) The rear wheel of a tractor has a 24 in radius. Find the angle (in radians) through which a wheel rotates in 11 sceonds if the tractor is traveling at a speed of 23 mph.

(17) One gear wheel turns another, the teeth being on the rims. The wheels have 40 cm and 50 cm radii, and the smaller wheel rotates at 10 rotations per minute. Find the angular speed of the larger wheel, in radians per second. (The answers is actually better if you don't bother with a calculator.)