Solutions to Review for Exam 4

MATH 1190

Sections Covered: 5.3, 5.4, 3.4, 4.1, 4.7

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) The volume of a cube is increasing at a rate of 1200 cm^3/min at the instant that its edges are 20 cm long? At what rate are the lengths of the edges changing at that instant? They are increasing at a rate of 1 cm/min.

(2) A stone dropped into a still pond sends out a circular ripple whose radius increases at a constant rate of 3 ft/sec. How fast is the area enclosed by the ripple increasing at the end of 10 sec?It is increasing at a rate of 180π sq. ft./sec.

(3) A 10 foot ladder rests against a wall that makes a right angle with the ground. A person slides the base of the ladder away from the wall at a rate of 6 in/sec. At the moment that the base is 6 feet from the wall, determine (a) the rate at which the top of the ladder is sliding down the wall, and (b) the rate at which the inclination angle between the base of the ladder and the ground is changing.(a) The top is falling at a rate of $3/8$ feet per sec. If y is the position of the top of the ladder (height from the ground), then at this instant $dy/dt = -3/8$. (b) The angle is decreasing at a rate of $1/16$ radian per second. If the angle is called θ, then at this instant $d\theta/dt = -1/16$.

(4) Explain why each statement below is false.

(a) If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x).$$

(b) If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$ for all $0 \leq x \leq 1$.

(c) If f is continuous on $[a, b]$, then f has a derivative on $[a, b]$.

(d) If f is continuous on $[a, b]$, then $\int_a^b x f(x)\,dx = \frac{x^2}{2} \int_a^b f(x)\,dx$.

Answers may vary to some extent. Here are some explanations:

(a) The definite integral $\int_a^b f(x)\,dx$ is a number; it's constant. If we define a function via this, it's derivative would be zero. This appears to be a garbled misstatement of the FTC part 1.

(b) There's no good reason to conclude that f is always zero. The graph could enclose a couple of regions of equal area with one each above and below the x-axis. Take the counter example with $f(x) = -2x + 1$.

(c) Differentiability implies continuity. But the converse isn't true. Take the standard counter example of $f(x) = |x|$ on $[-1, 1]$. It's continuous, but doesn't have a derivative defined on the interval—think about what happens at the origin.

(d) This is sad on so many levels. First, the definite integral on the left would be a number while the thing on the right would be a monomial cx^2. No variable factor can be *factored* out of an integral. Moreover, the integral of a product is not the product of the integrals.

(5) Evaluate the given definite integrals.

(a) given $\int_0^1 g(x)\,dx = 1,$ and $\int_0^2 g(x)\,dx = 7,$ evaluate $\int_1^2 g(x)\,dx = 6$

(b) $\int_{-1}^{2} (x^2+3x-1)\,dx = \frac{9}{2}$

(c) $\int_0^{\frac{\pi}{4}} \tan x \sec x\,dx = \sqrt{2} - 1$

(d) $\int_0^{\frac{\pi}{6}} \cos(r)\,dr = \frac{1}{2}$

(e) $\int_0^1 \frac{2}{1+x^2} = \frac{\pi}{2}$

(f) $\int_{-3}^{-2} \frac{1+x}{x}\,dx = \ln 2 - \ln 3 + 1$

(g) $\int_1^{\ln 3} e^x\,dx = 3 - e$

(h) $\int_1^4 \frac{x^3+8}{x^2}\,dx = \frac{27}{2}$

(i) $\int_0^1 (x^2+1)^2\,dx = \frac{28}{15}$

(j) $\int_{-1}^{1} 2^x \ln(2)\,dx = \frac{3}{2}$

(6) Evaluate each derivative.

(a) $\frac{d}{dx}\int_{-1}^{x} \frac{\sin t}{t^2+1}\,dt = \frac{\sin x}{x^2+1}$

(b) $\frac{d}{dx}\int_{x}^{2} te^{t^3}\,dt = -xe^{x^3}$

(c) $\frac{d}{dx}\int_{1}^{\sqrt{x}} \tan(t^2)\,dt = \frac{\tan(x)}{2\sqrt{x}}$

(d) $\frac{d}{dx}\int_{x}^{x^2} \sin^{-1}(t)\,dt = \frac{d}{dx}\left(\int_1^{x^2} \sin^{-1}(t)\,dt + \int_x^1 \sin^{-1}(t)\,dt\right) = 2x\sin^{-1}(x^2) - \sin^{-1}(x)$

(7) Find the average value of the function over the indicated interval.

(a) $f(x) = x^{2/3} \quad [-1, 1] \quad f_{avg} = \dfrac{3}{5}$

(b) $f(x) = x + \cos x \quad [0, \pi] \quad f_{avg} = \dfrac{\pi}{2}$

(c) $f(x) = 3^x \quad [0, 2] \quad f_{avg} = \dfrac{4}{\ln 3}$

(8) Find the point on the line $y = 5x + 4$ that is closest to the origin. The closest point is $\left(-\frac{10}{13}, \frac{2}{13}\right)$.

(9) A grain silo in the shape of a right circular cylinder is to be mounted on a concrete slab. The lateral surface and top will be made out of steel. The silo must hold 8π cubic meters of grain. Find the dimensions of the silo that will minimize the amount of steel used. The optimal silo will have radius $r = 2$ meters and also height $h = 2$ meters.

(10) A closed rectangular box with a square base is to be constructed. The material that the top and bottom (the square faces) are to be made of costs \$2 per square inch, and the material for the remaining four sides costs \$1 per square inch. If the box must have a volume of 16 cubic inches, determine the dimensions of the box that will minimize the cost of production. The optimal box will have dimensions 2" $\times$ 2" $\times$ 4".

(11) For each equation, identify a function whose root would be a solution to the equation. Set up Newton's Method for your function, and find x_1 using the given initial guess x_0.

(a) $\cos x = x, \quad x_0 = 0 \quad f(x) = \cos x - x, \quad x_1 = 1$

(b) $x^3 = x + 1, \quad x_0 = 1 \quad f(x) = x^3 - x - 1, \quad x_1 = \dfrac{3}{2}$