Review for Exam IV

MATH 2306

Sections Covered: 13, 14\(^1\), 15, 16, 17

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the Laplace transform using any method.

(a) \[ f(t) = e^{3t}(t-1)^2 \]
(b) \[ f(t) = t^2U(t-1)-e^{t}U(t-4) \]
(c) \[ f(t) = \begin{cases} 2t, & 0 \leq t < 3 \\ 1, & 3 \leq t \end{cases} \]

(2) Find the inverse Laplace transform using any method.

(a) \[ F(s) = \frac{s}{s^2 - 4s + 10} \]
(b) \[ F(s) = \frac{2s + 5}{(s - 3)^2} \]
(c) \[ F(s) = \frac{3e^{-2s}}{s(s + 1)^2} \]

(3) Solve the IVP using the Laplace transform.

(a) \[ y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 4 \]
(b) \[ y'' + 3y' - 4y = 80t, \quad y(0) = 1, \quad y'(0) = -4 \]
(c) \[ y'' + 4y' + 4y = 42e^5e^{-2t}, \quad y(0) = 1, \quad y'(0) = 0 \]

\(^1\)Sections 13 and 14 topics are integrated into section 16 problems.
(4) Solve the IVP using the Laplace transform.

\[ y'' + y = \mathcal{H}(t - \frac{\pi}{4}), \quad y(0) = 0, \quad y'(0) = 2 \]

(5) An LRC series circuit has inductance 1 h, resistance 2 ohms and capacitance 0.1 f. The initial charge on the capacitor and current in the circuit are \( q(0) = i(0) = 0 \). At time \( t = 0 \), a unit pulse voltage is applied to the circuit so that the charge satisfies

\[ L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C}q = \delta(t). \]

The function \( \delta(t) \) satisfies \( \mathcal{L}\{\delta(t)\} = 1 \). Find the charge on the capacitor \( q \) for \( t > 0 \) using the method of Laplace transforms.

(6) Suppose \( f \) is a function such that \( f(0) = 1 \) and \( \mathcal{L}\{f'(t)\} = \frac{\ln s}{s} \). Determine \( \mathcal{L}\{f(t)\} \). (In the words of Dennis Zill, “Don’t think deep thoughts.”)

(7) Find the Fourier series of the given function

(a) \( f(x) = 1 , \quad -\pi < x < \pi \)

(b) \( f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2x, & 0 \leq x < 2 \end{cases} \)

(c) \( f(x) = \begin{cases} -x - 1, & -1 < x < 0 \\ 1 - x, & 0 \leq x < 1 \end{cases} \)