Review for Exam IV

MATH 2306

Sections Covered: 13, 14[1], 15, 16, 17

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the Laplace transform using any method.

(a) $f(t) = e^{3t}(t-1)^2$

(b) $f(t) = t^2\mathscr{U}(t-1) - e^t\mathscr{U}(t-4)$

(c) $f(t) = \begin{cases} 2t, & 0 \le t < 3 \\ 1, & 3 \le t \end{cases}$

(2) Find the inverse Laplace transform using any method.

(a) $F(s) = \dfrac{s}{s^2 - 4s + 10}$

(b) $F(s) = \dfrac{2s + 5}{(s-3)^2}$

(c) $F(s) = \dfrac{3e^{-2s}}{s(s+1)^2}$

(3) Solve the IVP using the Laplace transform.

(a) $y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 4$

(b) $y'' + 3y' - 4y = 80t, \quad y(0) = 1, \quad y'(0) = -4$

(c) $y'' + 4y' + 4y = 42t^5e^{-2t} \quad y(0) = 1, \quad y'(0) = 0$

[1] Sections 13 and 14 topics are integrated into section 16 problems.

(4) Solve the IVP using the Laplace transform.

$$y'' + y = \mathscr{U}\left(t - \frac{\pi}{4}\right), \quad y(0) = 0, \quad y'(0) = 2$$

(5) An LRC series circuit has inductance 1 h, resistance 2 ohms and capacitance 0.1 f. The initial charge on the capacitor and current in the circuit are $q(0) = i(0) = 0$. At time $t = 0$, a unit pulse voltage is applied to the circuit so that the charge satisfies

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = \delta(t).$$

The *function* $\delta(t)$ satisfies $\mathscr{L}\{\delta(t)\} = 1$. Find the charge on the capacitor q for $t > 0$ using the method of Laplace transforms.

(6) Suppose f is a function such that $f(0) = 1$ and $\mathscr{L}\{f'(t)\} = \dfrac{\ln s}{s}$. Determine $\mathscr{L}\{f(t)\}$. (In the words of Dennis Zill, "Don't think deep thoughts.")

(7) Find the Fourier series of the given function

(a) $f(x) = 1, \quad -\pi < x < \pi$

(b) $f(x) = \begin{cases} 0, & -2 < x < 0 \\ 2x, & 0 \leq x < 2 \end{cases}$

(c) $f(x) = \begin{cases} -x - 1, & -1 < x < 0 \\ 1 - x, & 0 \leq x < 1 \end{cases}$