Review for Exam IV

MATH 2306 sections 51 & 54

Sections Covered: 7.2, 7.3, 11.2, 11.3

This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.

(1) Find the Laplace transform using any method.

(a) $f(t) = e^{3t}(t-1)^2 \qquad \mathscr{L}\{f(t)\} = \dfrac{2}{(s-3)^3} - \dfrac{2}{(s-3)^2} + \dfrac{1}{s-3}$

(b) $f(t) = t^2\mathscr{U}(t-1) - e^t\mathscr{U}(t-4) \qquad \mathscr{L}\{f(t)\} = \dfrac{2e^{-s}}{s^3} + \dfrac{2e^{-s}}{s^2} + \dfrac{e^{-s}}{s} - \dfrac{e^{4}e^{-4s}}{s-1}$

(c) $f(t) = \begin{cases} 2t, & 0 \le t < 3 \\ 1, & 3 \le t \end{cases} \qquad \mathscr{L}\{f(t)\} = \dfrac{2}{s^2} - \dfrac{2e^{-3s}}{s^2} - \dfrac{5e^{-3s}}{s}$

(2) Find the inverse Laplace transform using any method.

(a) $F(s) = \dfrac{s}{s^2-4s+10} \qquad \mathscr{L}^{-1}\{F(s)\} = e^{2t}\cos(\sqrt{6}t) + \dfrac{2}{\sqrt{6}}e^{2t}\sin(\sqrt{6}t)$

(b) $F(s) = \dfrac{2s+5}{(s-3)^2} \qquad \mathscr{L}^{-1}\{F(s)\} = 2e^{3t} + 11te^{3t}$

(c) $F(s) = \dfrac{3e^{-2s}}{s(s+1)^2} \qquad \mathscr{L}^{-1}\{F(s)\} = 3\mathscr{U}(t-2) - 3e^{-(t-2)}\mathscr{U}(t-2) - 3(t-2)e^{-(t-2)}\mathscr{U}(t-2)$

(3) Solve the IVP using the Laplace transform.

(a) $y'' - 2y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 4 \qquad y = 2e^t\cos(2t) + e^t\sin(2t)$

(b) $y'' + 4y' + 4y = 42t^5e^{-2t} \quad y(0) = 1, \quad y'(0) = 0 \qquad y = t^7e^{-2t} + 2te^{-2t} + e^{-2t}$

(4) Solve the IVP using the Laplace transform.

$$y'' + y = \mathscr{U}\left(t - \frac{\pi}{4}\right), \quad y(0) = 0, \quad y'(0) = 2$$

$$y = \mathscr{U}\left(t - \frac{\pi}{4}\right) - \cos\left(t - \frac{\pi}{4}\right)\mathscr{U}\left(t - \frac{\pi}{4}\right) + 2\sin t$$

(5) Find the Fourier series of the given function.

$$f(x) = \begin{cases} 0, & -1 < x < 0 \\ 2x, & 0 \le x < 1 \end{cases}$$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2((-1)^n - 1)}{n^2\pi^2}\cos(n\pi x) + \frac{2(-1)^{n+1}}{n\pi}\sin(n\pi x)$$

(6) Without actually computing either half range series, produce a plot of the graph of three periods on the interval $(-3p, 3p)$ of (a) the half range cosine series, and (b) the half range sine series of the given function.

$$f(x) = 4 - x^2, \quad 0 < x < 2$$ See last page.

(7) Find (a) the half range sine series and (b) the half range cosine series for f.

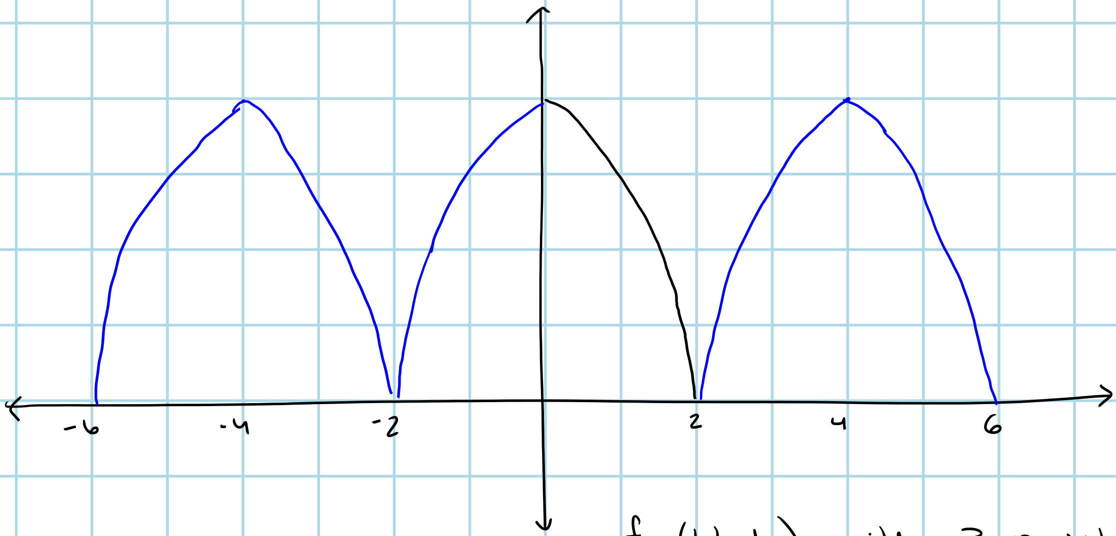
$$f(x) = \begin{cases} 1, & 0 < x < 1 \\ 2 - x, & 1 \le x < 2 \end{cases}$$

$$\text{(a)} \quad f(x) = \sum_{n=1}^{\infty} \left[\frac{2}{n\pi} + \frac{4\sin\left(\frac{n\pi}{2}\right)}{n^2\pi^2}\right]\sin\left(\frac{n\pi x}{2}\right)$$

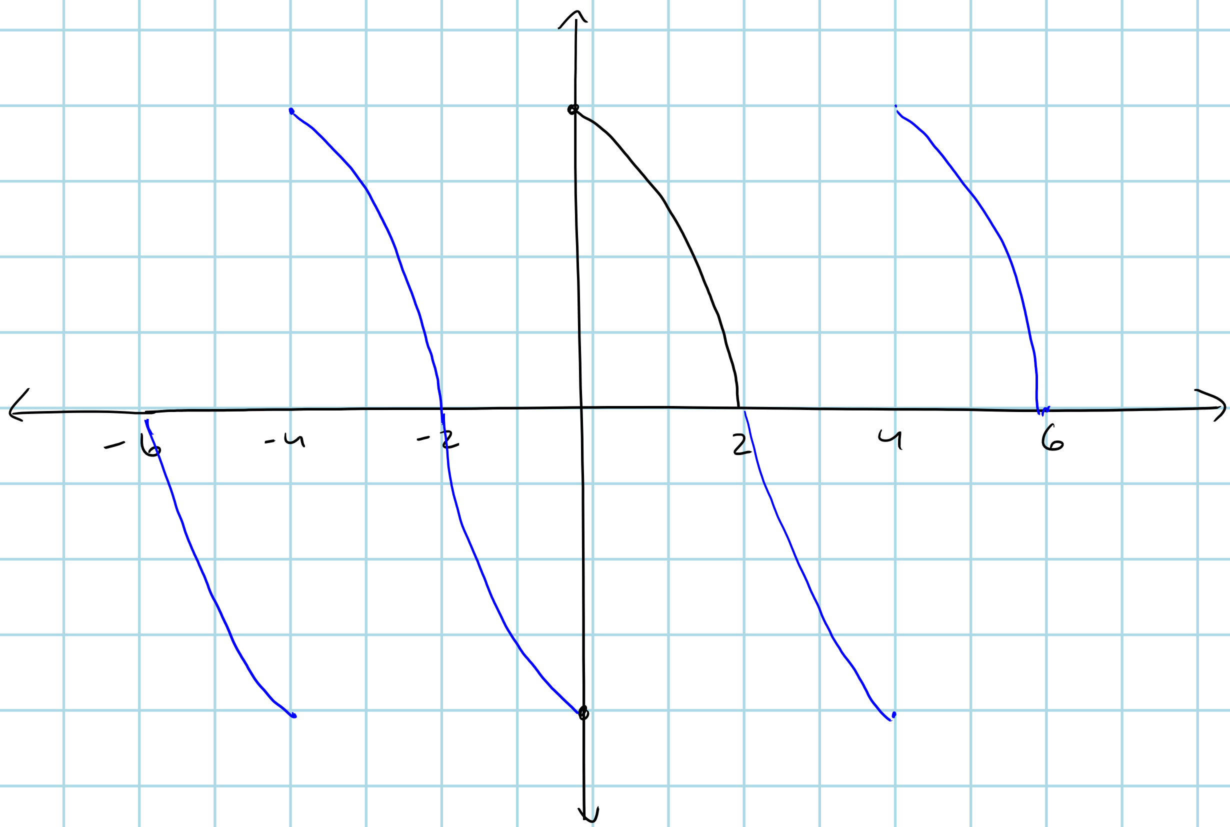
$$\text{(b)} \quad f(x) = \frac{3}{4} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\left[\cos\left(\frac{n\pi}{2}\right) - (-1)^n\right]\cos\left(\frac{n\pi x}{2}\right)$$

(8) Find the Fourier series of

$$f(x) = \begin{cases} -x - 1, & -1 < x < 0 \\ 1 - x, & 0 \le x < 1 \end{cases} \qquad f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\sin(n\pi x)$$



f (black) with 3 periods of cosine series (blue)



f (black) with three periods of its sine series (blue)