

## Additional Review for the Final

### MATH 2306 (Ritter)

The final exam will be comprehensive. This contains a review of section 16.

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) Solve the IVP using the Laplace transform.

(a)  $y'' + 4y = 1$   $y(0) = 0$ ,  $y'(0) = -1$

(b)  $y'' - y = 2 \cos(5t)$   $y(0) = 0$ ,  $y'(0) = 0$

(c)  $y'' - 2y' + 5y = 0$ ,  $y(0) = 2$ ,  $y'(0) = 4$

(d)  $y'' + 4y' + 4y = 42t^5 e^{-2t}$   $y(0) = 1$ ,  $y'(0) = 0$

(2) Solve the IVP using the Laplace transform.

$$y' - 7y = f(t), \quad y(0) = 0 \quad \text{where} \quad f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 2, & t \geq 1 \end{cases}$$

(3) Solve the IVP using the Laplace transform.

$$y'' + y = \mathcal{U}\left(t - \frac{\pi}{4}\right), \quad y(0) = 0, \quad y'(0) = 2$$

(4) Note that differentiating with respect to  $s$  inside the integral produces the formula

$$\frac{d}{ds} F(s) = \int_0^{\infty} \left( \frac{d}{ds} e^{-st} \right) f(t) dt = \int_0^{\infty} e^{-st} (-t f(t)) dt$$

That is, if  $F(s) = \mathcal{L}\{f(t)\}$ , then  $\mathcal{L}\{t f(t)\} = -F'(s)$ . Use this new rule along with the table of transforms to compute each transform

(a)  $\mathcal{L}\{t \sin t\}$

(b)  $\mathcal{L}\{t \cos(3t)\}$

(c)  $\mathcal{L}\{t e^t \cos(t)\}$