**Additional Review for the Final**

**MATH 2306 (Ritter)**

The final exam will be comprehensive. This contains a review of section 16.

*This review is provided as a courtesy to give some idea of what material is covered. Nothing else is intended or implied.*

(1) Solve the IVP using the Laplace transform.

(a) $y''+4y=1 \quad y(0)=0, \quad y'(0)=-1$ $\qquad y=\frac{1}{4}-\frac{1}{4}\cos(2t)-\frac{1}{2}\sin(2t)$

(b) $y''-y=2\cos(5t) \quad y(0)=0, \quad y'(0)=0$ $\qquad y=\frac{1}{26}e^{t}+\frac{1}{26}e^{-t}-\frac{1}{13}\cos(5t)$

(c) $y''-2y'+5y=0, \quad y(0)=2, \quad y'(0)=4$ $\qquad y=2e^{t}\cos(2t)+e^{t}\sin(2t)$

(d) $y''+4y'+4y=42t^5e^{-2t} \quad y(0)=1, \quad y'(0)=0$ $\qquad y=t^7e^{-2t}+2te^{-2t}+e^{-2t}$

(2) Solve the IVP using the Laplace transform.

$$y'-7y=f(t), \quad y(0)=0 \quad \text{where} \quad f(t)=\begin{cases} t, & 0\le t<1 \\ 2, & t\ge 1 \end{cases}$$

$$y=-\frac{1}{49}-\frac{1}{7}t+\frac{1}{49}e^{7t}-\frac{6}{49}\mathscr{U}(t-1)+\frac{1}{7}(t-1)\mathscr{U}(t-1)+\frac{6}{49}e^{7(t-1)}\mathscr{U}(t-1)$$

(3) Solve the IVP using the Laplace transform.

$$y''+y=\mathscr{U}\left(t-\frac{\pi}{4}\right), \quad y(0)=0, \quad y'(0)=2$$

$$y=\mathscr{U}\left(t-\frac{\pi}{4}\right)-\cos\left(t-\frac{\pi}{4}\right)\mathscr{U}\left(t-\frac{\pi}{4}\right)+2\sin t$$

(4) Note that differentiating with respect to $s$ inside the integral produces the formula

$$\frac{d}{ds}F(s) = \int_0^\infty \left(\frac{d}{ds}e^{-st}\right) f(t)\,dt = \int_0^\infty e^{-st}(-tf(t))\,dt$$

That is, if $F(s) = \mathscr{L}\{f(t)\}$, then $\mathscr{L}\{tf(t)\} = -F'(s)$. Use this new rule along with the table of transforms to compute each transform

(a) $\mathscr{L}\{t\sin t\} = \dfrac{2s}{(s^2+1)^2}$

(b) $\mathscr{L}\{t\cos(3t)\} = \dfrac{s^2-9}{(s^2+9)^2}$

(c) $\mathscr{L}\{te^t\cos(t)\} = \dfrac{(s-1)^2-1}{((s-1)^2+1)^2}$