Math 2306 Practice - Fourier Series

Names:

(1) Use the identity \( \sin(A) \cos(B) = \frac{1}{2} \left[ \sin(A + B) + \sin(A - B) \right] \) to show that for any positive integers \( m \) and \( n \)
\[
\int_{-\pi}^{\pi} \sin(mx) \cos(nx) \, dx = 0
\]

(2) Consider the function \( f(x) = \begin{cases} 
1, & -\pi < x < 0 \\
x, & 0 \leq x < \pi 
\end{cases} \)
(a) Find the average value\(^1\) of \( f \) on \((-\pi, \pi)\) (the integral of \( f \) divided by the length of the interval).

\(^1\)Note that this average value is equal to \( \frac{a_0}{\pi} \) that appears in the Fourier series of \( f \).
(b) Still working with this same function, \( f(x) = \begin{cases} 
1, & -\pi < x < 0 \\
x, & 0 \leq x < \pi 
\end{cases} \), find \( a_n \) and \( b_n \) and write out the Fourier series of \( f \). (You already found \( \frac{a_0}{2} \); don’t redo it.)